MAXIMAL SUBGROUPS OF $PSp_4(2^n)$ CONTAINING CENTRAL ELATIONS OR NONCENTERED SKEW ELATIONS[1]

BY

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1. Introduction

In [6] and [7], we laid the foundation for determining the maximal subgroups of $PSp_4(2^n)$. The purpose of this paper is to determine those maximal subgroups which contain either central elations or noncentered skew elations. Central elations are induced by transvections in $Sp_4(2^n)$, and noncentered skew elations are the duals of central elations. Other than the full symplectic groups over smaller fields, the maximal subgroups under consideration fall into six conjugacy classes, which are paired off by duality under the outer automorphism of $PSp_4(2^n)$.

The basic notation is that of [6] and [7]. By the Duality Theorem in [7], we need only look at subgroups of $PSp_4(q)$ which contain central elations. Repeated use will be made of the Center-Axis Theorem in [7]. See Huppert [12, pp. 191–214] for a discussion of the groups on a line. We will use I to denote the identity transformation or any identity matrix of appropriate rank.

THEOREM. *Let (V, f) be a nondegenerate, four-dimensional symplectic space over $GF(q)$, where $q = 2^n$; let δ be a duality on the incidence structure $PT(V, f)$ of points and totally isotropic lines.*

(i) *If G is a proper, superprimitive subgroup of $PSp_4(q)$ which contains a central elation, then G is the orthogonal group $GO(Q)$ for some nonmaximal index quadratic form Q on (V, f).*

(i*) *If G is a proper, superprimitive subgroup of $PSp_4(q)$ which contains a noncentered skew elation, then G is the dual $GO(Q)^\delta$ of the orthogonal group $GO(Q)$ for some nonmaximal index quadratic form Q on (V, f).*

COROLLARY. *The conjugacy classes of those maximal subgroups of $PSp_4(2^n)$ which contain central elations or noncentered skew elations are as follows:*

(a) *stabilizer of a point,*

(a*) *stabilizer of a totally isotropic line,*

(b) *maximal index orthogonal group,*

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