THE DYER-LASHOF ALGEBRA AND THE Λ-ALGEBRA

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Introduction

The Dyer-Lashof algebra R is an algebra of operations which act on the homology of infinite loop spaces. The algebra Λ may be considered as an algebra of operations which act on the homotopy of simplical restricted Lie algebras. The purpose of this paper is to describe the relation between R and Λ. As an application, we use this relation, together with the Adams spectral sequence, to obtain information about possible spherical classes in $H_*(\Omega^n S^n)$.

For each integer $i \geq 0$, there is a Kudo-Araki operation Q^i which acts on the mod-2 homology of each infinite loop space. The Dyer-Lashof algebra R is the free associative algebra over Z_2 generated by the Q^i, modulo the ideal of relations which hold in every infinite loop space. There are two types of relations: (1) Q^i is 0 when applied to a homology class of dimension greater than i, and (2) Adem-type relations which hold among iterates of the Q's. The structure of R is known from the work of Araki-Kudo, Browder, Dyer-Lashof, Madsen, May, and Nishida. The properties of R that we use are summarized in Section 1. In particular, certain iterates of the Q's (those which are called allowable of non-negative excess) form a basis for the vector space R. Let $\Omega^\infty S^\infty$ be the component containing the constant map of the space $\lim_n \Omega^n S^n$. The mod-2 homology of $\Omega^\infty S^\infty$ is a polynomial algebra with generators in 1-1 correspondence with the allowable basis elements of positive excess of R.

The algebra Λ is obtained (in [6]) as the homotopy of the free simplical restricted Lie algebra on one generator. Λ is shown to be the free associative algebra generated by certain elements λ_i, as $i = 0, 1, 2, \ldots$, modulo an ideal which turns out to be the same as the ideal of Adem relations for R. Not only is the algebraic structure of R similar to that of Λ, but, as we shall show, the action of the Steerod algebra and higher operations on Λ is related to the differential ∂ on Λ.

For each space X, the (unstable) Adams spectral sequence $\{E_r(X)\}$, $r = 1, 2, \ldots$, is a sequence of differential groups, which, roughly speaking, goes from the homology of X to the homotopy of X. Here we use the methods of Bousfield and the author [5], (modifications of those of Massey-Peterson [12]), to obtain the Adams spectral sequence for $\Omega^\infty S^\infty$. The term $E_1(\Omega^\infty S^\infty)$, defined by means of $H_*(\Omega^\infty S^\infty)$ and Λ, is shown to be itself isomorphic to Λ. This isomorphism is *not* filtration preserving, *nor* differential respecting. The precise formulation of this isomorphism (Lemma (5.1)) is the basis of our calculations. We then show (in Sections 6 and 7) that, except for elements related

Received January 18, 1974.