FINITE GROUPS WITH A QUASISIMPLE COMPONENT OF TYPE $PSU(3, 2^n)$ ON ELEMENTARY ABELIAN FORM

BY

PETER LANDROCK[1]

It is a quite common phenomenon among sporadic simple groups that some involution has a centralizer with a quasisimple component of even characteristic which is on elementary abelian form. By this we mean that the centralizer of the component has an elementary abelian Sylow 2-subgroup. (For definition of component, quasisimple etc., we refer the reader to for example D. Gorenstein's survey article on finite simple groups.) Examples of such sporadic simple groups are: Janko's first group J_1 ($Z_2 \times PSL(2, 4)$), the Mathieu group M_{12} ($Z_2 \times S_5$), the Hall-Janko group J_2 ($Z_2 \times Z_2 \times PSL(2, 4)$), the sporadic Suzuki group Su ($Z_2 \times Z_2 \times PSL(3, 4)$), Held's group He (a central extension of $PSL(3, 4)$ by $Z_2 \times Z_2$), Rudvalis' group Ru ($Z_2 \times Z_2 \times Sz(8)$), Conway's group Co_1 ($Z_2 \times Z_2 \times G_2(4)$) and Fischer's new simple group $F_2(?)$ ($Z_2 \times Z_2 \times F_4(2)$).

This gives rise to several classification problems, among which is the following natural one.

Classify finite (in particular simple) groups with an involution whose centralizer C is isomorphic to the direct product of an elementary abelian 2-group E and a group B containing a normal subgroup B_0 which is quasisimple of Bender-type such that $C_B(B_0) = Z(B_0)$.

However, to deal with this problem we need an additional assumption on the involutions of E. A natural one, at least when B_0 is of Bender-type, seems to be that C is the centralizer of all the involutions in E (trivially satisfied when $|E| = 2$.) This is a type of problem which for instance occurs in a recent work by D. Mason, in which he considers finite simple groups all of whose components are of Bender-type (and the centralizer of some involution not 2-constrained of course). Furthermore, J_2 and Ru satisfy this assumption.

Exactly this problem has been considered in the following cases when B_0 is isomorphic to one of the simple groups $PSL(2, q)$ or $Sz(q)$, $B = B_0$ and G is simple: $E \simeq Z_2$ and $B \simeq PSL(2, 2^n)$, by Z. Janko, $B \simeq PSL(2, 2^n)$, by F. L. Smith, $B \simeq Sz(q)$, by U. Dempwolff, and some as special cases in related problems which have been dealt with by M. Aschbacher and K. Harada.

Here we shall answer the question completely for all groups with B_0 quasisimple of $PSU(3, 2^n)$-type, the third class of groups of Bender-type.

Received January 11, 1974.

[1] Part of this work was supported by The Royal Society, London.