L^1-ALGEBRAS ON SEMIGROUPS

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The purpose of the paper is to extend the theory of L^1-algebras on semigroups. We first consider a finite product of idempotent intervals, with Lebesgue measure. Then we extend the theory to products of intervals with a group. In the next stage, all homomorphic images of the aforementioned type semigroups are examined. Finally, in the last section, we consider some examples. One example which comes readily out of this is a generalization of Bergman's work [2] on algebraically irreducible semigroups S in which S' is an idempotent thread (see Example 4.2).

1. Products of idempotent intervals

Let $I = \{a, b\}$ denote an interval of real numbers from a to b, where a or b may be infinite and I may or may not contain either of the endpoints a and b. With multiplication on I given by $xy = \max(x, y)$, and the natural topology, the space I is a topological semigroup. For any $E \subseteq I$, and point $x \in I$, define the set Ex^{-1} by

$$Ex^{-1} = \{y \in I \mid xy \in E\}.$$

Then $M(I)$ is the space of all bounded, regular, Borel measures on I equipped with total variation norm and multiplication given by

$$\mu * \nu(E) = \int_I \int_I \chi_E(xy)\, d\mu(x)\, d\nu(y) = \int_I \mu(Ey^{-1})\, d\nu(y),$$

where χ_E is the characteristic function of E. If m is Lebesgue measure on I, define $L^1(I, m)$ to be the space of all measures in $M(I)$ which are absolutely continuous with respect to m.

Lardy, in [5], has shown that $L^1(I, m)$ is a closed subalgebra of $M(I)$. Define $(a, b]$ as follows: $(a, b]$ is the ordinary open-closed interval if $b \neq \infty$, and if $b = \infty$, $(a, b]$ is obtained by "compactifying the right half" of $(a, \infty]$. In other words, $(a, \infty] = (a, \infty) \cup \{\infty\}$, where a neighborhood base of ∞ is given by $\{(c, \infty) \cup \{\infty\} \mid c \in (a, \infty)\}$. With this definition, Lardy has identified the maximal ideal space of $L^1(I, m)$ with the interval $(a, b]$. In addition, he has shown that $L^1(I, m)$ is regular, possesses approximate identities, and satisfies a Herglotz-Bochner theorem.

All through this discussion, S will denote the semigroup $\prod_{n=1}^{N} I_n$ (which we will write as $\prod I_n$ when the indexing is understood), where $I_n = \{a_n, b_n\}$ and

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