A CHARACTERIZATION OF PROJECTIVE SPACES IN TERMS OF h-ENCLOSABILITY[1]

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Introduction

All manifolds considered shall be closed and connected in the category $\mathscr{C}$ = Diff or PL. Following the notation and terminology of [1] we say that a $\mathscr{C}$ – manifold L^r is *h-enclosable* with a point A, and write $M^n = [L^r, A]$ (mod $\mathscr{C}$) when L^r is a $\mathscr{C}$ – submanifold of M^n with $M^n - L^r$ contractible onto A and for which L^r is a deformation retract of $M^n - A$. For example, when $F = R, C, Q$ or H we have $FP^n = [FP^{n-1}, A]$ (mod $\mathscr{C}$) where FP^n is the projective space over F. The object of this paper is to prove that projective spaces are to a large extent characterized by this property of h-enclosability. More precisely, the following result will be proved.

THEOREM 1. *Suppose* $M^n = [L^r, A]$ (mod $\mathscr{C}$) *where* $\mathscr{C} = PL$ *or* Diff *and* A *is a single point.*

(A) *If either one of* M^n *or* L^r *is not orientable then* $M^n \sim RP^n$ (*homotopy equivalence*) *and* $L^r \sim RP^r$ *and* $r = n - 1$.

(B) *If both* M^n *and* L^r *are orientable then:*

(i) M^n *is a homotopy sphere and* L^r *is a single point, if* n *is odd.*

(ii) *If* n *is even, then the only possible values for* r *are* 0, $n - 2$, $n - 4$, *and* $n - 8$. *If* $r = 0$, M^n *is a homotopy sphere. If* $r = n - 2$, $M^n \sim CP^{n/2}$ *and* $L^r \sim CP^{r/2}$. *If* $r = n - 4$ (*resp.* $r = n - 8$), M^n *is a cohomology* $QP^{n/4}$ (*resp.* $HP^{n/8}$) *and* L^r *is a cohomology* $QP^{r/4}$ (*resp.* $HP^{r/8}$).

1. Cohomology of M^n

Throughout this paper A denotes a point and we write $X \sim Y$ to mean that X is homotopically equivalent to Y. We write $M^n = [L^r, A]$ (mod $\mathscr{C}^+$) to indicate that $M^n = [L^r, A]$ (mod $\mathscr{C}$) and that both M^n and L^r are orientable. Though we state results for Diff and PL we will give detailed proofs only in the case of Diff. For the PL case we have only to replace the normal bundle of L in M as it occurs in an argument by the regular neighborhood of L in M. Throughout this paper we will be making repeated use of the following result proved in [2].

PROPOSITION 2. *Let* M^n, L^r *be closed connected manifolds and* $x \in M^n$. *If* $M^n - x \sim L^r$ *then* $r/n = l/(l + 1)$ *for some integer* $l \geq 0$.

From now on $\mathscr{C}$ will stand for Diff or PL.

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