ON THE HIERARCHY OF W. KRIEGER

BY

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In his paper "On ergodic flows and the isomorphism of factors" W. Krieger introduces a hierarchy $\Delta(n)$, $n \in \mathbf{N}$, labelling different weak equivalence classes of ergodic transformations of type III_0. The aim of the present paper is to answer a question of W. Krieger, namely to prove the existence of a weak equivalence class of type III_0 not in the above hierarchy. There is a close link between this hierarchy and the discrete decomposition $M = W^*(\theta, N)$ of factors of type III_0 [2, part V]. In fact in such a decomposition the restriction of θ to the center of N is unique, up to an induction on a non-zero projection in the sense of Kakutani [2, Theorem 5.4.2]. In particular the weak equivalence class of this restriction is uniquely associated to M. Starting from a weak equivalence class τ we get a factor M by the group measure space construction, hence if τ is of type III_0 we can associate to it the derived weak equivalence class τ' corresponding to discrete decompositions of M. A weak equivalence class τ belongs to the hierarchy if and only if $\tau^{(n)}$ fails to be of type III_0 for some n.

We compute the discrete decomposition of a large class of infinite tensor product of type I factors. In fact we show that any of the automorphisms T_p of W. Krieger [9, p. 87] which are strictly ergodic, appear as θ/Center of N in the discrete decomposition of some infinite tensor product of type I factors. Also we produce a weak equivalence class τ of transformation T_p of type III_0 such that $\tau' = \tau$ and hence not belonging to the above hierarchy.

We shall need some standard notations:

(1) Let $(k_i)_{i=1,2\dots}$ be a sequence of integers, $X_i = \{n, 1 \leq n \leq k_i\}$ a totally ordered set with k_i elements for each $i \in \mathbf{N}$, and $p = (p_i)_{i \in \mathbf{N}}$ a sequence of probability measures, p_i on X_i for each $i \in \mathbf{N}$. Then, as in [9, p. 87] we define an automorphism T_p of the measure space $X = \prod_{i=1}^{\infty} (X_i, p_i)$ by setting, for $x = (x_i)_{i \in \mathbf{N}} \in X$,

$$\begin{aligned} I(x) &= \min \{i \in \mathbf{N}, x_i < k_i\}, \\ (T_p(x))_i &= 1 \qquad \text{if} \quad i < I(x) \\ &= x_i + 1 \quad \text{if} \quad i = I(x) \\ &= x_i \qquad \text{if} \quad i > I(x). \end{aligned}$$

(2) Let $\{\lambda_{\nu,j}\}_{j=1,\dots,n_\nu,\nu\in\mathbf{N}}$, be an eigenvalue list, i.e., for each ν, λ_ν is a probability measure on a set E_ν with n_ν elements. Then for each ν we let M_ν be

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