ON FINITE LINEAR GROUPS OF DEGREE 16

BY

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1. Introduction

The main result of this paper is:

THEOREM 1. *Let G be a finite group with a faithful irreducible complex representation of degree* 16. *Then if P is a Sylow p-subgroup of G for $p \geq 19$ and Z is the center of G, either $P \lhd G$ or $p = 31$ and $G/Z \approx PSL_2(31)$.*

Theorem 1 has several consequences bearing on the situation of a group with a complex representation of degree smaller than a prime dividing its order. We state them here, using the same notation as above.

THEOREM 2. *Let p be a prime. Assume that $|P| = p$, $P \ntriangleleft G$, and G has a faithful irreducible complex representation of degree $d < p - 1$. Let $t|N: C| = p - 1$, where N, C are the normalizer, resp. centralizer, of P in G. If $t \geq 3$ then $t \geq 8$.*

It is known that if $t \geq 3$ then $t \geq 6$ [13]. In view of [2], Theorem 1 eliminates the only remaining numerical case when $t = 6$, namely $p = 19$ and $d = 16$. This case was also listed as unresolved in [1, Section 8] as $p = 19$, $d = 16$, $e = 3$.

THEOREM 3. *Assume $p > 7$. Let G have a faithful irreducible complex representation of degree $d < \max\{(7p + 1)/8, p + (3/2) - (p + 5/4)^{1/2}\}$. Then either $P \lhd G$ or $G/Z \approx PSL_2(p)$ and $d = (p \pm 1)/2$.*

For the exceptions to Theorem 3 when $p \leq 7$ see [9, Section 8.5] (or Theorem 4 below for the cases $d < p - 1$).

THEOREM 4. *Assume G has a faithful irreducible complex representation of degree $d \leq 27$. Suppose p is a prime, $p > d + 1$. Then one of the following must occur:*

(i) $P \lhd G$;

(ii) $G/Z \approx PSL_2(p)$, $d = (p \pm 1)/2$;

(iii) $p = 17$, $d = 15$, $G \approx SL_2(16) \times A$ *where A is abelian;*

(iv) $p = 7$, $d = 4$, *and* $G/Z \approx A_7$;

(v) $p = 5$, $d = 3$, *and* $G/Z \approx A_6$.

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