EXOTIC CONVERGENCE OF THE EILENBERG-MOORE SPECTRAL SEQUENCE

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Let $p: E \to B$ be a fibration over a connected space B with fiber F. The Eilenberg-Moore spectral sequence of p is a second quadrant spectral sequence which tries and sometimes fails to converge strongly to the homology of F (see [5]). The purpose of this paper is to determine what the spectral sequence *does* converge to. An abstract answer (Theorem 1.1) is that the spectral sequence almost always converges to the homology of the fiber of the nilpotent completion of the map p. A concrete answer (Theorem 2.1) is that under certain natural conditions on B and certain finiteness hypotheses the spectral sequence converges weakly to the homology of F with the $\pi_1(B)$ filtration.

As an aid to understanding these theorems, recall the similar behavior of the mod q Adams spectral sequence of a spectrum X. In absolute generality the spectral sequence converges only to the homotopy groups of some completion of X [1]. However, if X is connected and suitable finiteness conditions are satisfied, the spectral sequence converges to the actual homotopy groups of X with the "power of q" filtration. Note also that the spectral sequence converges *strongly* to the homotopy of X only in the rare case that each $\pi_i X$ is a q-group of finite exponent.

Sections 3 and 4 are devoted to applications of the convergence theorems in Section 1 and Section 2. In Section 3 we compute, in a certain sense, the homology of the universal cover of the nilpotent completion of a space X. In Section 4 we show that the cohomology of certain nilpotent groups is generated, in the sense of matric Massey products, by classes of degree one.

Throughout Section 1 and Section 2 we will work with the fixed fibration p described above. R will be a ring of the form $\mathbf{Z}/q\mathbf{Z}$ (q prime) or a subring of the rationals, and A will be a fixed R-module. We will freely use the ideas and conventions of [5]. In particular, we will associate to the fibration p a certain augmented cosimplicial space $F \to \mathbf{F}$, called the *Eilenberg-Moore object* of p. The mod A Eilenberg-Moore spectral sequence of p is understood to be the homotopy spectral sequence of the augmented tower of fibrations $A \otimes F \to \{Tot_s A \otimes \mathbf{F}\}_s$.

1. An abstract computation

Recall that for any space X, and any ring such as R, Bousfield and Kan ([2]) construct a functorial augmented tower of fibrations $X \to \{R_s X\}_s$. According to Dror ([4]), it is natural to think of this *tower* as the R-completion of X. For

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