MEASURES WHOSE POISSON INTEGRALS ARE PLURIHARMONIC II

BY

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1. Introduction

Let V be a vector space over $\mathbf{C}$ of complex dimension n with an inner product. If x and y are in V, then we will denote by $\langle x, y \rangle$ the inner product of x and y. We will denote by B the class of all x in V such that $\langle x, x \rangle < 1$, by $\bar{B}$ the class of all x in V such that $\langle x, x \rangle \leq 1$, and by S the class of all x in V such that $\langle x, x \rangle = 1$. We recall that the Poisson kernel of B is the function $\beta: \bar{B} \times B \to (0, \infty)$ defined by

$$\beta(x, y) = [(1 - \langle y, y \rangle)/(1 - \langle x, y \rangle)(1 - \langle y, x \rangle)]^n.$$

(We remark that β is the Poisson kernel with respect to the Bergman metric on B and not the Euclidean metric.)

If Y is a locally compact Hausdorff space, then we will denote by $M_+(Y)$ the class of all Radon measures on Y. Thus if $\mu \in M_+(Y)$ and $E \subset Y$, then $\mu(E) \geq 0$. We will denote by $M(Y)$ the complex linear span of those μ in $M_+(Y)$ for which $\mu(Y) < \infty$. (Thus if Y is compact, then $M(Y)$ is the complex linear span of $M_+(Y)$.) We recall that if X and Y are sets, if f is a function defined on the Cartesian product $X \times Y$, and if $(s, t) \in X \times Y$, then f_s and f^t are the functions defined on Y and X respectively by $f_s(y) = f(s, y)$ and $f^t(x) = f(x, t)$.

If $\mu \in M(S)$, then we define $\mu^\#: B \to \mathbf{C}$ by $\mu^\#(y) = \int \beta^y \, d\mu$. Thus $\mu^\# \in C^\infty(B)$. We will denote by σ the Radon measure on S which assigns to each open subset of S its Euclidean volume divided by the Euclidean volume of S (for the purpose of defining σ we regard S as the Euclidean sphere of real dimension $2n - 1$). Thus $\sigma(S) = 1$.

There is the following question.

1.1. If $\mu \in M(S)$, if $\mu^\#$ is pluriharmonic, and if $n \geq 2$, then do we have $\mu \ll \sigma$?

The purpose of this paper (which is a sequel to [2]) is to state and prove Theorem 3.15 and Corollary 4.7 which bear on the question 1.1. The results of the paper [2] suggest that the answer to the question 1.1 is yes. Theorem 3.15 and Corollary 4.7 of this paper support this suggestion.

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