ON PRIMITIVE PERMUTATION GROUPS WHOSE STABILIZER OF A POINT INDUCES $L_2(q)$ ON A SUBORBIT

BY

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1. Introduction

In the following we consider primitive permutation groups G acting on a finite set Ω. If $\alpha \in \Omega$ then G_α has a suborbit $\Delta(\alpha)$ such that the group $G_\alpha^{\Delta(\alpha)}$ induced on $\Delta(\alpha)$ is isomorphic to $L_2(q)$ and $|\Delta(\alpha)| = q + 1$, where $q \geq 4$ and $q = p^n$, p a prime. We state:

THEOREM. *Suppose G satisfies the above conditions then either*

(a) $G_\alpha \simeq L_2(q)$ *or*

(b) $p > 2$ *and* $G_\alpha \simeq L_2(q) \times Y$ *where Y is isomorphic to the normalizer of a S_p-subgroup in $L_2(q)$.*

The proof of the theorem will follow to a great extent the pattern of the work of C. C. Sims [9]. In this way we get bounds for $|G_\alpha|$ and structural informations of G_α. Then we use results about irreducible $F_p[L_2(q)]$-modules. In the case $p = 2$ also "2-local arguments" will enter. The notation is standard (see [4] and [14]).

2. Preliminary lemmas

In this section we collect some—mostly known—results, which will be used repeatedly.

PROPOSITION 2.1 (Walter, also see [1]). *Let G be a finite group having abelian S_2-subgroups. Then G possesses a normal subgroup H of odd index, such that*

$$H/O(H) \simeq X_0 \times X_1 \times \cdots \times X_n$$

where X_0 is an abelian 2-group and X_i $(1 \leq i \leq n)$ are finite simple groups isomorphic to $L_2(q)$, q suitable, or of type "Janko-Ree" (for the definition of type "Janko-Ree" see [1]).

PROPOSITION 2.2 (Gilman, Gorenstein [2]). *Let G be a finite simple group and $S \in Syl_2(G)$. Suppose* cl $(S) = 2$. *Then G is isomorphic to one of the following groups:*

$$L_2(q),\ q \equiv 7, 9 \ (\text{mod } 16),\ A_7,\ Sz(2^n),\ U_3(2^n),\ L_3(2^n),\ \textit{or}\ PSp(4, 2^n).$$

Received January 15, 1975.