SPECTRAL DECOMPOSITION AND DUALITY

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Introduction

The purpose of this paper is to improve our previous result [3] concerning the duality of decomposable operators. In that paper we have proved that the dual of a 2-decomposable operator is also 2-decomposable. We shall prove here that the dual of a 2-decomposable operator is actually decomposable. This result has some interesting consequences. The first one is that on a reflexive Banach space, any 2-decomposable operator is decomposable, thus improving a result contained in [1] and answering positively a question raised in [4]. A second one is that the dual of any decomposable operator is a decomposable operator. A similar result for a more restrictive notion of decomposability was obtained in [5]. Some other consequences are related to the quasinilpotent equivalence of 2-decomposable operators.

The paper consists of four sections. In Section 1 we give some definitions and auxiliary results. In Section 2 we prove a general decomposition theorem for continuous linear functionals which will be used essentially in the proof of our main theorem and which seems to be interesting by itself. Finally, Section 3 contains the main result of the paper, and Section 4, its consequences.

1. Preliminaries

We begin by recalling some definitions from the theory of spectral decompositions. Let X be a complex Banach space and $L(X)$ be the space of all continuous linear operators on X.

DEFINITION 1. [2], [4] (a) An operator $T \in L(X)$ is said to be *m-decomposable* (m is a natural number, $m \geq 2$) if for every finite covering $\{G_1, \ldots, G_k\}$ of the spectrum $\sigma(T)$ of T consisting of $k \leq m$ open sets, there exist k maximal spectral subspaces $Y_1, \ldots, Y_k$ of T such that:

(i) $X = \sum_{j=1}^{k} Y_j$,

(ii) $\sigma(T \mid Y_j) \subset G_j$ $(1 \leq j \leq k)$.

(b) T is said to be *decomposable* if it is m-decomposable for every number m.

A *maximal spectral subspace* Y of T is a (closed linear) subspace invariant for T, and containing any other invariant subspace with a smaller spectrum (i.e., $TZ \subset Z$ and $\sigma(T \mid Z) \subset \sigma(T \mid Y)$ imply $Z \subset Y$).

It is easy to see that some results proved in [2] for decomposable operators remain valid for 2-decomposable operators. Thus, denoting the resolvent of T, by $R(\cdot\,; T)$, for any $x \in X$, the analytic function $z \to R(z; T)x$ defined on the resolvent set, $\rho(T)$, has a single-valued maximal extension. We denote by

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