ERGODIC ACTIONS WITH GENERALIZED DISCRETE SPECTRUM

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The investigation of extensions in the theory of ergodic actions of locally compact groups was undertaken by the author in [26]. In particular, we considered the notion of extensions with relatively discrete spectrum, and saw how the classical von Neumann-Halmos theory of transformations with discrete spectrum could be generalized to the case of extensions. In this paper, which is a sequel to [26], we study those actions which can be built up from a point by taking extensions with relatively discrete spectrum and inverse limits. We shall say that such actions have generalized discrete spectrum.

A similar construction is well known in topological dynamics. In [4], Furstenberg introduced the notion of an isometric extension of a continuous transformation group, and called an action quasi-isometric if it could be built up from a point by taking isometric extensions and inverse limits. The main result of [4] is the striking theorem that among the minimal transformation groups, the quasi-isometric ones are precisely those that are distal. Thus, one obtains a description of the structure of an arbitrary minimal distal transformation group, and using this, one can answer a variety of questions about such groups.

The structure of extensions with relatively discrete spectrum was described in Theorem 4.3 of [26]. Examination of the conclusion of this theorem shows that extensions with relatively discrete spectrum are a reasonable measure-theoretic analogue of Furstenberg's isometric extensions. Thus, we can consider actions with generalized discrete spectrum as a measure-theoretic analogue of the quasi-isometric transformation groups. Parry has described, at least for actions of the integers, a measure-theoretic analogue of the topological notion of distallity [20]. It is not difficult to generalize Parry's definition to arbitrary group actions, and now the question arises as to whether one can prove a measure-theoretic analogue of Furstenberg's theorem. We prove such a theorem below. It asserts that among the nonatomic ergodic actions, those with a separating sieve (as Parry called his actions) are precisely those with generalized discrete spectrum. Using this theorem, one sees immediately, for example, that any minimal distal action preserving a probability measure has generalized discrete spectrum.

Though there are formal similarities between the proof of our theorem and Furstenberg's proof, the proofs are basically quite different. Our proof depends upon, among other things, generalizing the concepts of weak mixing and the

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