θ-PERFECT AND θ-ABSOLUTELY CLOSED FUNCTIONS

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1. Introduction

Given a function $f: X \to Y$, a class $\mathscr{A}$ of topological spaces, and a class $\mathscr{C}$ of functions, the extension function problem is to find a topological extension Z of X and extension function $F: Z \to Y$ of f such that $Z \in \mathscr{A}$ and $F \in \mathscr{C}$. The first author [5] proved that if f is continuous, then it is possible to construct a continuous perfect extension F on some topological extension Z of X and noted that if X and Y are Hausdorff spaces, then Z is not necessarily Hausdorff. Viglino [21] started with a continuous function f and Hausdorff spaces X and Y and required that F be continuous and Z be Hausdorff; he showed it was possible to obtain a maximal continuous extension function F (maximal in the sense that there is no proper continuous extension of F defined on a Hausdorff topological extension). Such maximal continuous extensions are called absolutely closed and are characterized in [6] in terms of a closedness-like property of F and a compactness-like property of point-inverse of F—analogous to the perfect function setting.

The notion of "θ-continuity" between Hausdorff spaces is more useful in certain cases (cf., [8], [11], [13], [16]) than "continuity." In this paper, we start with a θ-continuous function f and Hausdorff spaces X and Y and require that F be θ-continuous and Z be Hausdorff. Maximal θ-continuous extension functions are called θ-absolutely closed and are investigated in Section 4; in particular, a θ-continuous function between H-closed spaces is θ-absolutely closed. We are able to show that if Y is regular or H-closed, Urysohn, then f has a θ-absolutely closed extension F. A concept stronger than θ-absolutely closure, called θ-perfect, is developed in Section 3 and characterized in terms of a closedness-like property and compactness-like property of point-inverses.

A θ-continuous function between Hausdorff spaces that has a θ-continuous extension between their Katětov extensions is called a θ-p-map and is studied in Section 5. Now θ-p-maps are related to θ-absolutely closed functions as every θ-p-map from a Hausdorff space into an H-closed space has a θ-absolutely closed extension. In Section 6, the compactness-like properties of point-inverses of θ-perfect and θ-absolutely closed functions are investigated and related. A filter concept, called almost convergence, is defined in this section, developed in Section 2, and used to obtain many of the results in the rest of the paper.

The reader is referred to [3] for the definitions not given here. Here are a few additional definitions and results that are needed throughout the paper.

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