ZETA FUNCTIONS OF SELBERG'S TYPE FOR COMPACT SPACE FORMS OF SYMMETRIC SPACES OF RANK ONE

BY

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0. Introduction

Let M be a compact Riemann surface of genus $g \geq 2$. Then $M = \Gamma \backslash H$ where H is the upper half plane, and Γ is a discrete subgroup of $SL(2, \mathbf{R})$, acting freely on H via fractional linear transformations. Let T be a finite dimensional unitary representation of Γ with character χ. In a well-known paper [21], A. Selberg showed how we may attach a zeta function $Z_\Gamma(s, \chi)$ (of a complex variable s) to this data, and showed how the location and the orders of the zeros of Z_Γ give us information about the spectrum of M on the one hand and about the topology of M (via its Euler characteristic) on the other hand.

Now let G be a connected semisimple Lie group with finite center, K a maximal compact subgroup, and H the symmetric space G/K; We endow H with a G-invariant metric. Let Γ be a discrete torsion-free subgroup of G such that $\Gamma \backslash G$ is compact. Then the manifold $\Gamma \backslash H (\Gamma \backslash G/K)$ which we will call M, is a compact Riemannian manifold, whose simply connected covering manifold is H, and we have $\Gamma \cong \pi_1(M)$. M is a compact space form of H.

We assume throughout this paper that rank $(G/K) = 1$.

Let T be a finite-dimensional unitary representation of Γ, and let χ be its character. The object of this paper is to study a certain zeta function $Z_\Gamma(s, \chi)$ attached to the data (G, K, Γ, χ). We shall see that this zeta function has all of the properties possessed by Selberg's zeta function. The following properties will be discovered:

(1) Z_Γ is holomorphic in a half plane $\operatorname{Re} s > 2\rho_0$ where ρ_0 is a positive real number depending only on (G, K).

(2) Z_Γ has a meromorphic continuation to the whole complex plane.

(3) Z_Γ satisfies the functional equation

$$Z_\Gamma(2\rho_0 - s, \chi) = \left\{ \exp \left(\kappa \chi(1) \operatorname{vol}(\Gamma \backslash G) \int_0^{s - \rho_0} c(it)^{-1} c(-it)^{-1} \, dt \right) \right\} Z_\Gamma(s, \chi).$$

Here, vol $(\Gamma \backslash G)$ is the volume of $\Gamma \backslash G$ in a suitable normalization, κ is a positive integer depending only on (G, K), and $c(\cdot)$ is Harish-Chandra's c-function which appears in the Plancherel measure for G/K [10]. In our case, this function is essentially a function of one complex variable.

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