EVERY PLANAR MAP IS FOUR COLORABLE
PART I: DISCHARGING

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1. Introduction

We begin by describing, in chronological order, the earlier results which led to the work of this paper. The proof of the Four Color Theorem requires the results of Sections 2 and 3 of this paper and the reducibility results of Part II. Sections 4 and 5 will be devoted to an attempt to explain the difficulties of the Four Color Problem and the unusual nature of the proof.

The first published attempt to prove the Four Color Theorem was made by A. B. Kempe [19] in 1879. Kempe proved that the problem can be restricted to the consideration of “normal planar maps” in which all faces are simply connected polygons, precisely three of which meet at each node. For such maps, he derived from Euler's formula, the equation

\[ 4p_2 + 3p_3 + 2p_4 + p_5 = \sum_{k=7}^{k_{\text{max}}} (k - 6)p_k + 12 \]

where \( p_i \) is the number of polygons with precisely \( i \) neighbors and \( k_{\text{max}} \) is the largest value of \( i \) which occurs in the map. This equation immediately implies that every normal planar map contains polygons with fewer than six neighbors.

In order to prove the Four Color Theorem by induction on the number \( p \) of polygons in the map (\( p = \sum p_i \)), Kempe assumed that every normal planar map with \( p \leq r \) is four colorable and considered a normal planar map \( M_{r+1} \) with \( r + 1 \) polygons. He distinguished the four cases that \( M_{r+1} \) contained a polygon \( P_2 \) with two neighbors, or a triangle \( P_3 \), or a quadrilateral \( P_4 \), or a pentagon \( P_5 \); at least one of these cases must apply by (1.1). In each case he...