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STABILITY OF GAUSS MAPS

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1. Introduction

In this article we prove that certain geometrical properties of immersions of differentiable surfaces M (without prescribed metric) into Euclidean space $\mathbf{E}^3$ are generic. Our results concern properties related to the set of *parabolic* points (points where the Gaussian curvature, of the metric induced by the immersion, vanishes). Observe that all the interesting properties (in the differentiable category) of the Gauss normal map occur in a neighborhood of this set. In some sense our results form an extension of a small, but significant, part of Feldman's (et. al.) work on geometric transversality (see for example [3]–[7], [12], [13]). Although we could use much of the machinery and many of the constructions in these works, there seems to be little advantage to doing so in the low-dimensional case at hand. In fact, we discovered the results before reading these papers. We are however indebted for the idea of applying transversality theory to questions of geometrical genericity. Before stating our main result, we introduce some definitions and notation.

Let $C^\infty(M, N)$ be the set of all C^∞ maps from a compact manifold M to an arbitrary manifold N. We give $C^\infty(M, N)$ the topology of uniform convergence of each k-jet ($k = 0, 1, 2, \ldots$). Given an open subset $S \subset C^\infty(M, N)$, we call a property of maps in S *generic* if the subset of maps in S having that property is open and dense in S. We are mostly concerned with the case where M is a compact, orientable surface, N is $\mathbf{E}^3$, and $S = I(M, \mathbf{E}^3)$ is the set of immersions of M into $\mathbf{E}^3$. We define the following properties (P1), (P2), and (P3) of maps $f \in I(M, \mathbf{E}^3)$.

(P1) The Gaussian curvature K from the metric induced on M by f has the property that K and dK do not vanish simultaneously. Hence the parabolic set ($K = 0$) consists of a finite disjoint collection of smoothly embedded circles, the normal derivative of the Gaussian curvature on these circles is nonzero, and none of these points are extrinsically planar: for $0 \neq dK = d(k_1 k_2) = k_1 dk_2 + k_2 dk_1$, where k_1 and k_2 are the principal curvatures at a parabolic point, which implies that k_1 and k_2 are not both zero.

(P2) Property (P1) holds and the zero principal curvature direction field (corresponding to the principal curvature which is zero) along the parabolic curves is transverse to those curves except at a finite number of points. At these points, the derivative of the angle α of transversality is nonzero as one moves

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