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REPRESENTING HOMOLOGY CLASSES BY EMBEDDED CIRCLES ON A COMPACT SURFACE

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Introduction

It is well known that a one-dimensional homology class on a torus T^2, that winds around one standard generator m times and the other standard generator n times can be represented by one embedded circle if and only if m and n are relatively prime. However, the classification of one dimensional homology classes that can be represented by an embedded circle on an arbitrary compact surface has until now been only partially resolved. We define a homology class in $H_1(M^2)$ to be primitive if the induced class in $H_1(M^2)$/torsion is the zero class or is not a nontrivial multiple of any other class. In [6], [7], and [8] it is shown that primitive classes on orientable surfaces and primitive or twice primitive classes on nonorientable surfaces are precisely the classes representable by embedded circles.

Although there are algorithms (see [2] and [10]) for determining which elements of the fundamental group $\pi_1(M^2)$ can be represented by embedded circles, they are too complicated to deal with the above classification. For orientable compact surfaces we have developed an explicit algorithm for representing a primitive class by one embedded circle. When combined with the Classification Theorem for Oriented Surfaces, this algorithm becomes a useful tool for studying certain classification problems on surfaces, which we shall discuss later. We would like to thank the referee for his suggested improvements of the paper. We now outline the algorithm.

Section 1

Let $\alpha_1, \beta_1, \ldots, \alpha_g, \beta_g$ be the standard generators of $H_1(M_g, Z)$ where M_g denotes a surface of genus g. Let c_i be the standard circle which disconnects M_g between hole i and hole $i + 1$.

If $\gamma = \sum_{i=1}^{g} m_i \beta_i + n_i \alpha_i$, then we will let $[m_1, n_1], \ldots, [m_g, n_g]$ denote γ. If $\gamma = [m_1, n_1]$ is a primitive class on the torus, then γ can be represented by an embedded circle of "constant slope" n_1/m_1. Actually, for our purposes it will be better to represent $\gamma = [m_1, n_1]$ on a torus as follows: First represent γ by m_1

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