ILLINOIS JOURNAL OF MATHEMATICS
Volume 22, Number 2, June 1978

COMPLEMENTS OF CODIMENSION-TWO SUBMANIFOLDS I: THE FUNDAMENTAL GROUP

BY

JUSTIN R. SMITH

Introduction and statement of results

This paper is the first in a series that will study the homotopy types of the complements of certain classes of codimension-two imbeddings of compact manifolds—in particular, this paper will study the groups that can occur as fundamental groups. The results of this paper apply equally to smooth, *PL*, or topological imbeddings and manifolds. All manifolds in this paper will be assumed to be *compact* and *connected* and all imbeddings will be assumed to be *locally-flat* and to carry the boundry of the imbedded manifold transversely to that of the ambient manifold.

This paper generalizes Kervaire's characterization of high-dimensional knot groups in [4].

The results in this paper formed part of my doctoral dissertation and I am indebted to my thesis advisor, Professor Sylvain Cappell, for having suggested this problem and for his guidance and criticism. I would also like to thank the referee for his helpful comments.

Before we can state the main result of this paper we need the following definition:

DEFINITION AND PROPOSITION 1. *Let M^m be a compact manifold and let*

$$w\colon \pi_1(M) \to \mathbf{Z}_2 = \{\pm 1\}$$

be a homomorphism and $\mathbf{Z}^w$ the $\mathbf{Z}\pi_1(M)$-module of twisted integers defined by w. If $x \in H^2(M, \mathbf{Z}^w)$ is any element, define

$$C(x, w) = \mathbf{Z}^w/(x \cap H_2(M; \mathbf{Z}\pi_1(M));$$

the cap product takes its values in $H_0(M; \mathbf{Z}^w \otimes \mathbf{Z}\pi_1(M)) = \mathbf{Z}^w$. If x' is the image of x under the change of coefficient homomorphism

$$H^2(M; \mathbf{Z}^w) \to H^2(M; C(x, w)),$$

then x' is in the image of the injection

$$H^2(\pi_1(M); C(x, w)) \to H^2(M; C(x, w))$$

Received April 26, 1976; received in revised form October 18, 1976.