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SECTIONAL REPRESENTATION OF MULTITOPOLOGICAL SPACES RELATIVE TO A FAMILY OF SMOOTHNESS CATEGORIES

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Introduction

The purpose of this paper is to study those multitopological spaces (a set with a family of topologies on it) that can be represented, relative to a family of differentiability classes, by an embedding of the set in a differentiable manifold. For example, if the family of differentiability classes is $\{C^r\}$, $r = 1, 2, \ldots$, then an embedding $A \subset E$ of a set A in a smooth manifold E is said to represent a sequence of topologies $\{\tau^r\}$, $r = 1, 2, \ldots$, on A if τ^r is coinduced by the family of all C^r-maps from the reals to E with image in A, for $r = 1, 2, \ldots$.

After this notion of representation is discussed in some detail, the problem of representing multitopological structures on a manifold by sections of a differentiable vector bundle is then studied. In particular, it is shown that those structures that are induced by a locally finite, decreasing sequence of regular, local kernels can be so represented. From this follows a generalized Whitney embedding theorem: namely, any decreasing sequence of foliation topologies on an n-manifold can be represented by an embedding in Euclidean $2n$-space. The case in which the foliations are trivial (leaf = manifold) reduces to the classical Whitney embedding theorem, while the case in which the foliations have points for leaves reduces to a generalized form of the construction of a continuous, nowhere differentiable function. The paper concludes with a discussion of some further problems.

The general procedure is to construct global representations by "pasting together" local ones. However, the usual technique of forming global sections from local ones by using a partition of unity does not work since sectional representations, in general, are not closed under addition and scalar multiplication. The key assumption is regularity (§5) since it allows one to build global representations from local ones by other means (5.1 and 5.2).

See [6], [7], and [9] for closely related topics.

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