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COHOMOLOGY OF FIBER SPACES IS REPRESENTABLE

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Introduction

In recent years interest in cohomology theories defined on a category of fiber spaces has increased. See for example [5], [17], and [22]. It is the purpose of this paper to show that such theories are representable by a suitable Ω-spectrum.

Sections I, II, and III are devoted to proving a fibered version of E. Brown's representability theorem as formulated in [7] and [13]. In Section IV we give axioms for a cohomology theory over B general enough to include sheaf cohomology, prestack cohomology, and group bundle cohomology. The main difference from other axiom systems as found in [5] or [11] is our weakening of the homotopy axiom. In Section V we define the notions of reduced cohomology theories and Ω-spectra over B. In this section we review Mielke's work on group bundle cohomology and conclude with the representability theorem mentioned before.

The category theory language we use is that of [20]. All spaces considered are of the homotopy type of a C.W. complex or pair and the base space B of the text is assumed to be Hausdorff. We do this since our main result depends on the fundamental theorems of J. H. C. Whitehead. That the various construction in the text do not take us outside the category of C.W.-spaces is a consequence of results found in [19].

Proofs have been omitted (for example 2.3) where they are just fibered versions of standard homotopy theory arguments.

Finally I wish to mention that as this paper was being prepared for publication I learned of the parallel work of Rolf Schön in [25]. Thus, the main result of Section V are independent achievements of Schön and myself by slightly different methods.

I. Preliminaries

Throughout this paper we let B be a fixed connected space.

We let $\downarrow B$ stand for the category of fiber spaces of B defined as follows. An object α of $\downarrow B$ is a triple $\alpha = (X_\alpha, A_\alpha, p_\alpha)$ where A_α is a closed subspace of X_α and p_α is a map from X_α to B. A morphism $f\colon \alpha \to \gamma$ of B is a map

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