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FUNCTIONS OF UNIT MODULUS ON BOUNDARY PORTIONS OF DOMAINS WITH A CERTAIN CIRCULAR SYMMETRY

BY

T. L. McCoy

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1. Introduction

Let Δ^N denote the unit ball in the space C^N of N complex variables, and consider functions f holomorphic in Δ^N. When $N=1$, the function $\log|f|$ can be prescribed almost arbitrarily on the boundary $\partial\Delta^N$. When $N>1$, however, the behavior of $|f|$ on smaller subsets of $\partial\Delta^N$ tends to be enough to determine f completely. For instance, if $|f|=1$ on an open subset of $\partial\Delta^N$ then f is constant. Recently Forelli ([2], Theorem 1.5) has shown that if f_1 and f_2 are holomorphic in Δ^N and continuous in the closure, with $|f_1|=|f_2|$ on an open subset of $\partial\Delta^N$, then in fact f_1/f_2 reduces to a constant.

In the present paper we will find that there are subsets $U\subset\partial\Delta^N$ which are topologically thinner than open sets, such that f is completely determined by the non-tangential limits of $|f|$ on U, under certain growth restrictions on f; we obtain a result which overlaps Forelli's but does not contain it. This is a consequence of Theorem B, stated in Section 2. Our Theorem C contains a result of Rudin (unpublished, cited in [2]) which states that if f is any non-constant inner function of $\Delta^N (N>1)$ then the cluster set of f at every boundary point of $\partial\Delta^N$ consists of the full unit disc.

The results of this paper concern not only Δ^N, but a rather wide class of domains containing Δ^N; the *slice domains* defined near the end of this introductory section.

In the remainder of this section we set out the notation and definitions to be used throughout. In Section 2 we state the main theorems, and discuss

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