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TOEPLITZ OPERATORS ON THE BALL WITH PIECEWISE CONTINUOUS SYMBOL

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1. Introduction

Let B denote the open unit ball in $\mathbf{C}^n$ and let $A^2(B)$ denote the Bergman space of square-integrable holomorphic functions on B. The Toeplitz operator with symbol φ, T_φ, is defined by $T_\varphi f = P(\varphi f)$, $f \in A^2(B)$, where φ is in $L^\infty(B)$, and P is the orthogonal projection of $L^2(B)$ onto $A^2(B)$. Let E be a $(2n-1)$-dimensional real hyperplane in $\mathbf{C}^n$ intersecting B. The set $B \setminus E$ then consists of two components, which we will label B_+ and B_-. We define

$$HC(B) = \{\varphi \in L^\infty(B)\colon \varphi \,|B_+, \varphi|\, B_- \text{ are uniformly continuous}\}.$$

The main purpose of this paper is to compute the essential spectrum of T_φ for φ in $HC(B)$, and to show, in particular, that it is connected.

Note that $HC(B)$ is a closed subalgebra of $L^\infty(B)$, and that we can write

$$HC(B) = \{\langle f, g\rangle\colon f \in C(\bar{B}_+),\, g \in C(\bar{B}_-)\}.$$

Our interest in $HC(B)$ stems from the fact that in many ways it seems like a reasonable analogue of the algebra PC on the unit circle T. Recall that PC is the closed subalgebra of $L^\infty(T)$ consisting of piecewise continuous functions on T, and that $\varphi^\#$ is the curve obtained by joining left- and right-hand limits of φ by a line segment at points of discontinuity. For details, and a proof of the following, see [3, pp. 20–23].

PROPOSITION 1.1. *If φ and ψ are in PC, then:*
(i) *$T_\varphi T_\psi - T_\psi T_\varphi$ is compact.*
(ii) *T_φ is Fredholm if and only if $\varphi^\#$ does not pass through the origin.*

Here T_φ denotes the Toeplitz operator acting on the Hardy space H^2 on the circle.

We will show that this proposition remains essentially true for φ and ψ in $HC(B)$. We point out that 1.1(i) depends on the fact that we can approach a point of discontinuity of a function from only two directions on T. This property is retained by the functions in $HC(B)$. For suppose λ is in E and $\varphi = \langle f, g\rangle$ is in $HC(B)$. If we approach λ through B_+, then $\lim \varphi(z) = f(\lambda)$, and if we approach λ through B_-, then $\lim \varphi(z) = g(\lambda)$.

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