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# ON THE FORMALITY OF $K-1$ CONNECTED COMPACT MANIFOLDS OF DIMENSION LESS THAN OR EQUAL TO $4K-2$

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In all that follows, coefficients will be assumed to be in the field of rational numbers $\mathbf{Q}$. All spaces will be at least simply connected and of finite type. The categories of differential graded algebras, coalgebras, and Lie algebras will be denoted by DGA, DGC, and DGLA, respectively, and objects in these categories will be referred to as algebras, coalgebras, and Lie algebras. Algebras and coalgebras will be assumed to be associative, cocummutative, and homologically connected.

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The following notions are due to Sullivan [2] [3], Neisendorfer [5] [6], and Baues-Lemaire [1].

DEFINITION. A simply connected algebra $M$ (respectively Lie algebra $L$) is called minimal if it is free and the differential is decomposable. A minimal model for a simply connected algebra $A$ (respectively Lie algebra $L'$) is a minimal algebra $M$ (respectively minimal Lie algebra $L$) and a weak equivalence $\rho: M \to A$ (respectively $\rho: L \to L'$).

Minimal models exist for simply connected algebras (respectively Lie algebras) and are unique up to isomorphism [3] [5] [6].

Let $\mathscr{E}(X)$ be the algebra of rational de Rham chains on a space $X$. We denote the minimal model for $\mathscr{E}(X)$ by $M_X$.

DEFINITION. A space $X$ is called formal if there is a weak equivalence

$$\rho: M_X \to H^*(M_X).$$

Since $M_X$ determines the homotopy type of $X$ [2], a formal space has its homotopy type determined by its cohomology algebra.

We can now state the major result of the paper. It is a generalization of a result found in [5].

THEOREM. *Let $X$ be a compact, $k-1$ connected manifold of dimension less than or equal to $4k-2$, $k>1$. Then $X$ is formal.*

Below we give a brief sketch of the theory necessary for the proof of this theorem. Readers desiring more detail are referred to [4], [5], and [6].

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