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THE TOTAL NEGATION OF A TOPOLOGICAL PROPERTY

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0. Introduction

The central theme in this paper is the uniform generation of new topological properties from old. Two of the best known properties obtained in this way are total disconnectedness (deriving from connectedness) and scatteredness (deriving from perfectness, i.e. having no isolated points). A third property, lesser known but interesting in its own right, is *pseudofiniteness* (the *cf-spaces* studied in [8], [9], [10], [12]) or the class of spaces whose compact subsets are finite. This last-mentioned property derives from compactness in the manner we will explore here.

In general, given a class K of topological spaces (K is closed under homeomorphism) we define the class Anti (K) in such a way that "totally disconnected" is co-extensive with "Anti (connected)" and so on. The "anti-property" of most interest to us here is pseudofiniteness which we henceforth relabel "anticompactness". We will also be interested in related anti-properties (Anti (sequentially compact), Anti (Lindelöf), etc.) but they will recieve secondary emphasis. The general behavior of the operation Anti $(\cdot)$ itself will occupy some of our attention. However at this stage there are many more questions than answers, so our general treatment will be sketchy, serving mainly to tie together ideas which otherwise may appear to be unrelated.

Our set-theoretic conventions are as follows: (i) ω_α denotes the αth infinite initial ordinal, where α is any ordinal. Since we assume the Axiom of Choice throughout, we identify ω_α with the cardinal $\aleph_\alpha$. $\omega = \omega_0$. (ii) An ordinal α is the set of its predecessors. Greek letters near the beginning of the alphabet will usually denote ordinals, while the letters κ, λ, μ will be reserved for cardinals. (iii) The ordinal successor of α is $\alpha+1=\alpha\cup\{\alpha\}$, the cardinal successor of κ is κ^+. (iv) If A is any set $P(A)$ denotes the power set of A. (v) B^A is the set of all maps $f\colon A \to B$. The cardinality of A is written $|A|$. (vi) If κ is a cardinal then $\exp(\kappa)=|2^\kappa|=|P(\kappa)|$. $\exp(\omega)$ is usually denoted by c. (vii) The cartesian product of a family $\langle A_i\colon i\in I\rangle$ of sets is denoted $\prod_I A_i$. If $A_i = A$ for all $i \in I$ then the set A^I will also at times be denoted $\prod_I (A)$. Further notations will be introduced as they arise in the discussion. The referee's kind suggestions regarding exposition are gratefully acknowledged.

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