ILLINOIS JOURNAL OF MATHEMATICS
Volume 23, Number 2, June 1979

REPRESENTING CODIMENSION-ONE HOMOLOGY CLASSES ON CLOSED NONORIENTABLE MANIFOLDS BY SUBMANIFOLDS

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In [5], Julie Patrusky and the author proved that a codimension-one homology class on a closed orientable connected piecewise linear manifold can be represented by a closed connected orientable submanifold precisely when the class is primitive. If M is a closed n-dimensional manifold, we will call a class in $H_{n-1}(M, Z)$ primitive if the induced class in $H_{n-1}(M, Z)$/torsion is the zero class or is not a nontrivial multiple of any other class.

The representation theorem we prove here is for closed connected nonorientable P.L. manifolds and its proof is much more involved than is the proof of the orientable case. In dimension two our theorem implies that an integer homology class on a connected closed nonorientable surface can be represented by an embedded circle if and only if the class is primitive or twice a primitive class.

Recall that the Universal Coefficient Theorem implies that if M is a closed, connected, n-dimensional P.L. manifold, then $H_{n-1}(M, Z) = Z_2 \oplus F$ where F is a free abelian group. After triangulation, an orientable k-dimensional P.L. submanifold naturally represents a class in $H_k(M, Z)$. (See [8].) We will call a closed oriented $(n-1)$-dimensional submanifold $N \subset M$ representing a class $\delta \in H_{n-1}(M, Z)$ a minimal representative for δ if there is no other submanifold representative for δ having fewer components. Let $|N|$ denote the number of components of N.

I would like to thank Larry Larmore for discussions on Theorem 2 and the referee who made very many comments on improving the paper.

THEOREM 1. *Suppose M is a closed connected nonorientable n-dimensional P.L. manifold. Let σ denote the order two class in $H_{n-1}(M, Z)$. If N is a minimal representative for a nonzero $\delta \in H_{n-1}(M, Z)$, then:*

(1) *If $M-N$ is not connected, then each nonorientable component of $M-N$ has one end.*

(2) *Every component of $M-N$ with three ends comes from cuts along two components of N.*

(3) *Each orientable component of $M-N$ has at most four ends. If there is a component of $M-N$ with four ends, then $M-N$ is connected and $|N| = 2$.*

Received May 4, 1977.

[1] This research was supported in part by a National Science Foundation grant.