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TOPOLOGICAL SPACES IN WHICH BLUMBERG'S THEOREM HOLDS II

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1. This note consists of some "odds and ends" involving Blumberg's theorem. Section 2 contains an example of a Baire space with a point countable base for which Blumberg's theorem does not hold; Section 3 deals with Blumberg's theorem for linearly ordered spaces; Section 4 is concerned with a strong form of Blumberg's theorem.

2. If X denotes a set, then $\mathbf{P}(X)$ denotes the collection of all subsets of X. If $A \subset X$ and $\mathcal{F} \subset \mathbf{P}(X)$, then $\mathcal{F} \cap A$ denotes $\{F \cap A; F \in \mathcal{F}\}$ and $\mathcal{F}^*$ denotes $\mathcal{F} \sim \{\emptyset\}$. If $(X, \mathcal{T})$ is a topological space, a subset $\mathcal{P}$ of $\mathcal{T}^*$ is called a pseudo-base for $\mathcal{T}$ if every element of $\mathcal{T}^*$ contains an element of $\mathcal{P}$. A collection of sets of called σ-disjoint if it is the union of a countable set of disjoint collections. The set of real numbers is denoted by R; the set of positive integers by N.

2.1. THEOREM. *If $(X, \mathcal{T})$ is a Baire space that has either a σ-point finite or σ-locally countable pseudo-base, then the following statement, known as Blumberg's theorem, holds for X.*

2.2. If φ is a real valued function defined on X, then there is a dense subset D of X such that $\varphi \mid D$ is continuous.

Proof. This follows from [15, Proposition 1.7] and the following statements.

2.3. If $\mathcal{T}$ has a σ-locally countable pseudo-base, then it has a σ-disjoint pseudo-base.

Proof. If $\mathcal{C}$ is a locally countable subset of $\mathcal{T}^*$ and $\mathcal{U}$ is a maximal disjoint subcollection of $\mathcal{T}^*$ such that $(\mathcal{C} \cap U)^*$ is countable for every U in $\mathcal{U}$, then $\mathcal{C}' = \cup\{(\mathcal{C} \cap U)^*: U \in \mathcal{U}\}$ is a σ-disjoint subcollection of $\mathcal{T}^*$ such that every element of $\mathcal{C}$ contains an element of $\mathcal{C}'$. ■

Remark. In [5, Theorem 2.1] it is shown that $\mathcal{T}$ has a σ-disjoint pseudobase whenever it has a σ-locally countable base.

2.4. PROPOSITION [6, Theorem 3.10]. *If $(X, \mathcal{T})$ is a Baire space and $\mathcal{C}$ is a point finite subset of $\mathcal{T}^*$, then there is a dense subset D of X such that $\mathcal{C}$ is locally finite at every point of D.*

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