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CHEVALLEY GROUPS AS STANDARD SUBGROUPS, III

BY
GARY M. SEITZ

Introduction

In this paper we complete the proof of the main theorem stated in [13]. This involves proving that a certain subgroup, G_0, of a group G is, in fact, normal in G.

Throughout the paper we let G be a finite group having a standard subgroup A such that $|Z(A)|$ is odd, $\tilde{A}=A/Z(A)$ is a group of Lie type of Lie rank at least 3 and defined over a field of characteristic 2, and $C_G(A)$ has cyclic Sylow 2-subgroups. Let t be an involution in $C_G(A)$ and assume $t \notin Z^*(G)$. In [13] we showed that with the assumption of a certain hypothesis (Hypothesis (*) in [13]) either $\tilde{A} \cong O^+(8,2)'$ and $G \cong \mathrm{Aut}\,(M(22))$, or there exists a t-invariant semisimple subgroup $G_0 \leq G$ such that $|Z(G_0)|$ is odd and $\tilde{G}_0 = G_0/Z(G_0)$ satisfies one of the following:

(i) $\tilde{G}_0 \cong \tilde{A} \times \tilde{A}$, the components interchanged by t.
(ii) $\tilde{G}_0$ is a group of Lie type defined over a field of characteristic 2 and t acts on $\tilde{G}_0$ as an outer automorphism.

Our goal in the present paper is to show $G_0 = E(G)$. We remark that our work here might be useful in the verification of Hypothesis (*) of [13]. In that hypothesis we assumed results concerning standard subgroups of type $PSp(6,2)$, $PSU(6,2)$, and $O^{\pm}(8,2)'$. However, these assumptions are only needed in establishing the existence of the group G_0. Therefore, when one considers standard subgroups of type $PSp(6,2)$, $PSU(6,2)$, or $O^{\pm}(8,2)'$ it will probably be sufficient to just construct an appropriate group G_0. The methods of this paper will yield $G_0 \trianglelefteq G$ once results analogous to those in §15 are verified.

It is in this part of the proof of the main theorem that we make use of the $B(G)$-conjecture, and here it is only used for the "wreathed case" (type (i)). With extra work and signalizer arguments it may be possible to work around the $B(G)$-conjecture. In the "quasisimple case" (type (ii) we will use a recent result of Holt [10].

Throughout the paper we keep the above notation. Also, we set $\Omega = \{G_0^g\colon g \in G\}$ and write $\alpha = G_0$. Our goal is to show that $\Omega = \{\alpha\}$. Notice that $G_\alpha = N_G(G_0)$. For $X \leq G$, let $\Delta(X)$ denote the set of fixed points of X on Δ.

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