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CHEVALLEY GROUPS AS STANDARD SUBGROUPS, II

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Introduction

This paper continues the work that was begun in [13]. Our situation is that A is a standard subgroup of a finite group G and $\tilde{A} = A/Z(A)$ is a group of Lie type having Lie rank at least 3 and defined over a field of characteristic 2. Our goal, in this paper, is to show that under the hypotheses of the main theorem of [13], either (a), (d), or (e) of that theroem holds, or there is an involution $t \in C_G(A)$ and a t-invariant subgroup, $G_0 \leq G$, such that G_0 satisfies (b) or (c) of the main theorem. Once we prove the existence of such a group G_0, all that will remain in the proof of the main theorem is the verification that $G_0 = E(G)$. That verification will occur in part three of the series.

Our construction of the group G_0 is as follows. Using the results of §4 of [13] we find a subgroup $X \leq A$ so that $O^{2'}(C_A(X))$ is a standard subgroup of $C_G(X)$ and $t \notin Z^*(C_G(X))$. By induction, Hypothesis $(*)$, or by appealing to the literature, we have the structure of $E = E(C_G(X))$. The group G_0 will be $\langle E, E^w \rangle$, where w is a suitable element of the Weyl group of A. The structure of G_0 is obtained by developing sufficient commutator information in order to apply the work of Curtis [5]. However, there are some difficulties in obtaining the necessary commutator relations. This is due, in part, to the fact that root subgroups of A may be properly contained in root subgroups of G_0, and in some cases not even contained in root subgroups of G_0. Another difficulty occurs when X is taken as an abelian Hall subgroup of a group, J, generated by two opposite root subgroups of A, and we find that J does not centralize $E(C_G(X))$.

Throughout the paper we operate under the following assumptions: $|Z(A)|$ is odd, $K = C_G(A)$ has cyclic Sylow 2-subgroups, and $\tilde{A} \not\cong Sp(6,2)$, $U_6(2)$, $O^{\pm}(8,2)'$, or $L_n(2^a)$. The omission of $\tilde{A} \cong L_n(2^a)$ is justified by the corollary in [14]. Let $R \in Syl_2(K)$ and $\langle t \rangle = \Omega_1(R)$.

5. Preliminaries

If X is any subgroup of G we set $X_A = \langle (O^{2'}(A \cap X))^X \rangle$. So $X_A \trianglelefteq X$. We will need a slight generalization of (1.3) of [14].

(5.1) *Let X be a finite group, P a standard subgroup of X with $C_X(P)$ of*

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