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MULTIPLIERS OF THE DIRICHLET SPACE

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Introduction

This paper deals with the space of analytic functions on the unit disc in the complex plane for which $\sum n^{\alpha}|a_n|^2$ is finite, where $\{a_n\}$ represent the Taylor coefficients and α is a real number. For $\alpha = 1$ this space can alternately be described by demanding that the Dirichlet integral $\iint |f'|^2\,dx\,dy$ be finite. We also consider real variable analogs of these spaces on the circle and on Euclidean space. In $\mathbf{R}^n$, these are the fractional Sobolev space L^p_α.

Our principal result is a characterization of the pointwise multipliers of these spaces. Various authors have studied properties of these multipliers and determined sufficient conditions, see [24], [5], [14], [15], [16], and in the complex case [20], [25].

Denote by D_α the space of analytic functions for which the norm

$$\left\{\sum_{n=0}^{\infty} (1+n^2)^{\alpha}|a_n|^2\right\}^{1/2}$$

is finite.

THEOREM A. *An analytic function $f(z)$ multiplies D_α $(0 < \alpha \le \frac{1}{2})$ if and only if f is bounded on $|z| < 1$ and there is a constant A such that*

$$\iint_{\cup S(I_j)} |f'|^2(1-|z|)^{1-2\alpha}\,dx\,dy \le A\,\mathrm{Cap}_\alpha(\cup I_j)$$

for all finite disjoint collections of subarc $\{I_j\}$ on the circle.

Here $S(I)$ denotes the "square" in the disc with side I and $\mathrm{Cap}_\alpha(\cdot)$ denotes an appropriate (Bessel) capacity depending on α. For $\alpha = \frac{1}{2}$ (Dirichlet space) the classical logarithmic capacity may be used.

For multipliers of $L^p_\alpha(\mathbf{R}^n)$ and also a boundary characterization on D_α we have the following result:

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