MULTIPLIERS OF THE DIRICHLET SPACE

BY

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Introduction

This paper deals with the space of analytic functions on the unit disc in the complex plane for which \( \sum n^a |a_n|^2 \) is finite, where \( \{a_n\} \) represent the Taylor coefficients and \( \alpha \) is a real number. For \( \alpha = 1 \) this space can alternately be described by demanding that the Dirichlet integral \( \int \int |f'|^2 \, dx \, dy \) be finite. We also consider real variable analogs of these spaces on the circle and on Euclidean space. In \( \mathbb{R}^n \), these are the fractional Sobolev space \( L^p \).

Our principal result is a characterization of the pointwise multipliers of these spaces. Various authors have studied properties of these multipliers and determined sufficient conditions, see [24], [5], [14], [15], [16], and in the complex case [20], [25].

Denote by \( D_\alpha \) the space of analytic functions for which the norm

\[
\left\{ \sum_{n=0}^{\infty} (1 + n^2)^\alpha |a_n|^2 \right\}^{1/2}
\]

is finite.

**THEOREM A.** An analytic function \( f(z) \) multiplies \( D_\alpha (0 < \alpha \leq \frac{1}{2}) \) if and only if \( f \) is bounded on \( |z| < 1 \) and there is a constant \( A \) such that

\[
\int \int_{S(I)} |f'|^2 (1 - |z|)^{1 - 2\alpha} \, dx \, dy \leq A \Cap_a (\cup I_j)
\]

for all finite disjoint collections of subarc \( \{I_j\} \) on the circle.

Here \( S(I) \) denotes the “square” in the disc with side \( I \) and \( \Cap_a (\cdot) \) denotes an appropriate (Bessel) capacity depending on \( \alpha \). For \( \alpha = \frac{1}{2} \) (Dirichlet space) the classical logarithmic capacity may be used.

For multipliers of \( L^p_a(\mathbb{R}^n) \) and also a boundary characterization on \( D_\alpha \) we have the following result:

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