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RELATION WITH THE HOPF INVARIANT REVISITED

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1. Introduction

The title of this note refers to Section 8 of Adams' paper *On the groups* $J(X)$, IV [4]. There Adams used his results from [2] to establish a formula which determines the mod p Hopf invariant in terms of the complex e-invariant (Proposition 8.2 [4] or Theorem 2 below). As an outgrowth of his 1970 lectures at Chicago [5], Adams reformulated the results of [2] in an article *Chern characters revisited* [1]. When related methods from the Chicago lectures are applied to a suitable version of the e-invariant, they yield a new proof of Proposition 8.2 which seems conceptually simpler. The object of this note, then, is to reformulate the e-invariant in a more general context and, with this and the Chicago "technology", revisit Proposition 8.2 in a spirit similar to the one with which Adams revisited his earlier Chern characters paper [2].

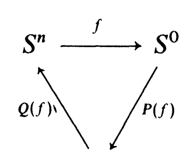
2. Definitions and statement of results

We begin by defining a homotopy invariant in a manner reminiscent of the definition of the invariant e_C which uses the Chern character [4; p. 41]. Let E be a ring spectrum with unit $i: S^0 \to E$ and let η_L and η_R respectively denote the homomorphisms

$$(E \wedge i)_*: \pi_*(E) = \pi_*(E \wedge S^0) \to \pi_*(E \wedge E)$$

$$\text{and} \quad (i \wedge E)_*: \pi_*(E) = \pi_*(S^0 \wedge E) \to \pi_*(E \wedge E).$$

Let $f \in \pi_n(S^0)$ be given and let



denote the associated cofiber triangle.

Now suppose that $f_*: E_*(S^n) \to E_*(S^0)$ is zero. (Let $\iota_k \in \pi_k(E \wedge S^k)$ denote the $E_*(S^0)$ generator $i \wedge S^k: S^0 \wedge S^k \to E \wedge S^k$.) As $f^*(\iota_0) = 0$, there is an extension

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