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LOCAL DILATIONS

BY

MARK D. MEYERSON[1]

1. Introduction

A local dilation is an embedding of a metric space which "stretches" in all small regions. The concept was introduced by the author in [6, p. 309] where Theorem 3.1 of this paper was proved. Local dilations were also used in [7], especially a special case of Corollary 3.4 below. These papers have applied local dilations by noting that a convergent sequence of local dilations, since close points are pushed apart, is likely to converge to a one-to-one function. Considered in the setting of elastic behavior, local expansions can be considered as locally stretching transformations, although the "strain" (see Fritz John [4]) may well be infinite.

In this paper we examine several properties of local dilations. We show that a local dilation from any closed manifold, with any "reasonable" metric, into itself is an isometry in the "path-metric". We show that a strictly starlike region in a hyperplane in E^n can be "pushed out" along a right cylinder (and not along a slanting cylinder) with a local dilation. Convexity properties and fixed point properties are considered. And we introduce and use the concept of path-metric. In addition, many counterexamples are included.

2. Basic properties

A *global dilation* is a continuous function (map) between metric spaces,

$$f\colon \langle X, \rho\rangle \to \langle Z, d\rangle,$$

such that for x and y in X, $d(f(x), f(y)) \geq \rho(x, y)$. (It's just the opposite of a contractive map.) A *local dilation* is an embedding between metric spaces, $h\colon X \to Z$, such that any point of X has a neighborhood N with $h \mid N$ a global dilation. An embedding is a map which is a homeomorphism when the target space is properly restricted.

LEMMA 2.1. *If $h\colon \langle X, \rho\rangle \to \langle Z, d\rangle$ is a local dilation and X is compact, then h is a uniform local dilation. In other words, there is a $\delta > 0$ such that for $x, y \in X$ with $\rho(x, y) < \delta$, we have $d(h(x), h(y)) \geq \rho(x, y)$.*

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