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UNIQUENESS THEOREMS FOR MINIMAL SURFACES

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Introduction

There are three basic uniqueness theorems in the theory of compact minimal surfaces in $\mathbf{R}^3$. The first theorem is due to Rado [19] and states that if γ is a Jordan curve which admits a one-to-one orthogonal or central projection on a convex plane curve, then γ bounds a unique minimal disk which is a graph over the plane. The second theorem due to Nitsche ([14] and also [5]) states that a smooth Jordan curve with total curvature less or equal to 4π bounds a unique minimal disk and the disk is immersed. A third theorem which follows from convergence properties for minimal disks is that any C^2 Jordan curve sufficiently C^2 close to a fixed plane C^2 Jordan curve bounds a unique minimal disk which is a graph over the plane (see [4] and [24]).

In this paper we generalize these three theorems to show that the curves above bound unique compact minimal surfaces. In the case of a Jordan curve with total curvature less than or equal to 4π, we need to make the additional assumption that the curve lies on the boundary of a convex set. It is still not known if this additional assumption is needed. The tools used in the proofs are the maximum principle for minimal surfaces, the geometric Dehn's lemma in [11], Nitsche's uniqueness theorem [14] and the manifold representation of smooth immersed minimal disks in $\mathbf{R}^3$ [24]. Except for Theorem 2, the results in this paper also appear in the author's book "Lectures on Plateau's Problem" [9] published by I.M.P.A. A generalization of Theorem 3 using a completely different approach also appears in [12].

1. Generalization of a theorem by Rado

The following well-known geometric inequality will be one of our basic tools for proving uniqueness theorems.

MAXIMUM PRINCIPLE. *Suppose M_1 and M_2 are graphs of two C^2 functions f_1, f_2: $D \to \mathbf{R}$ where*

(1) *D is the unit disk in $\mathbf{R}^2$,*
(2) *M_1 has zero mean curvature,*
(3) *M_1 and M_2 are tangent to the xy plane at the origin,*
(4) *$f_1(p) \le f_2(p)$ for all $p \in D$.*

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