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HYPERFUNCTIONS AS BOUNDARY VALUES OF GENERALIZED AXIALLY SYMMETRIC POTENTIALS

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1. Introduction

In the classical Dirichlet problem on the unit disk, one starts with a given function $f(\theta)$ on the unit circle and seeks a harmonic function $f(r, \theta)$ in the open unit disk that converges to $f(\theta)$ as r goes to 1. The solution to this problem is very well known. However, the solution to the converse problem i.e. finding a boundary function to which a given harmonic function in the interior of the disk converges, was found relatively recently by Gelfand [1], Johnson [4], Köthe [5] and Sato [6]. It turns out that this solution always exists in the space of hyperfunctions on the unit circle and is unique.

In [4] Johnson gives a characterization for the solutions of Laplace equation in the unit disk, namely, f is harmonic on the unit disk if and only if there is a sequence $\{g_n\}$ of continuous functions on the unit circle such that

$$\lim_{n\to\infty}(n!\|g_n\|_\infty)^{1/n} = 0$$

and

$$f(r, \theta) = \sum_{n=0}^{\infty} \frac{1}{2\pi}\int_0^{2\pi} p_r^{(n)}(\theta - t)g_n(t)\,dt$$

where $\|g_n\|_\infty = \sup_{0\leq t<2\pi}|g_n(t)|$ and $P_r^{(n)}(\theta)$ is the nth derivative of Poisson kernel for the unit disk. In addition to that, he shows that the space $\mathscr{H}^*$ of hyperfunctions on the unit circle is isomorphic to the space $\mathscr{F}$ of harmonic functions on the unit disk. The correspondence $f \leftrightarrow \tilde{f}$ where $f \in \mathscr{F}$ and $\tilde{f} \in \mathscr{H}^*$ is given by $f_r(\theta) = P_r * \tilde{f}$ where $*$ stands for the convolution, and $f_r(\theta) \to \tilde{f}$ in $\mathscr{H}^*$ as $r \to 1$.

More recently, Staples and Kelingos [7] have characterized all solutions of a perturbed Laplace equation in the unit disk by identifying their generalized boundary values.

In this paper we prove similar results for the regular solutions to the partial differential equation of Generalized Axially Symmetric Potentials (GASP):

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{2\mu}{y}\frac{\partial\phi}{\partial y} = 0 \quad \text{where } \mu > 0.$$

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