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POSITIVE OPERATORS ON SPACES OF BAIRE FUNCTIONS

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The symbol θ will be used to denote the zero element of any vector space. Suppose L is a Riesz space (lattice ordered vector space). For notation and basic terminology concerning Riesz spaces, the reader is referred to Luxemburg and Zaanen [6]. The sequence $f_1, f_2, f_3, \ldots$ of L is said to be *order Cauchy* if there exists a sequence $y_1 \geq y_2 \geq \cdots \geq \theta$, $\bigwedge y_n = \theta$ such that, for $m \geq n$, $|f_m - f_n| < y_n$. If every order Cauchy sequence converges, then L is *order Cauchy complete*. It is *order separable* if each subset with a supremum has a countable subset with the same supremum. Also, L is *almost σ-complete* if it is Riesz isomorphic to a subspace $L^{\sim}$ of a σ-complete space M with the property that if $m \in M^+$, there is a sequence $\theta \leq u_1 \leq u_2 \leq \cdots$, $u_n \in L^{\sim}$ such that $\bigvee u_n = m$. In particular, if L is order separable, it is almost σ-complete (Aliprantis and Langford [2]). The Riesz space L is *universally complete* if it is complete and every disjoint subset of L^+ has a supremum.

Suppose X is a set and Ω is a collection of real valued functions defined on X. Then $B_1(\Omega)$ (the first Baire class of Ω) is the set of all pointwise limits of sequences of Ω, $B_2(\Omega) = B_1(B_1(\Omega))$, and in general if α is an ordinal, $\alpha > 0$, $B_\alpha(\Omega)$ is the family of pointwise limits of sequences from $\bigcup_{\alpha > \gamma} B_\gamma(\Omega)$. If ω_1 is the first uncountable ordinal then $B_{\omega_1}(\Omega) = B_{\omega_1 + 1}(\Omega)$ which will be denoted $B(\Omega)$. For a discussion of Baire spaces see Mauldin [7] or [8].

Let LS Ω (lower semi-Ω) be the set of pointwise limits of non-decreasing sequences from Ω, US Ω be the set of pointwise limits of non-increasing sequences in Ω, and Ω^* be the set of bounded functions in Ω.

Spaces of the form $B(\Omega)$ include the set of all A measurable functions for some σ-algebra A and the σ-laterally complete function spaces as discussed in Chapter 7 of Aliprantis and Burkinshaw [1].

A complete ordinary function system Ω is a linear lattice of functions containing the constants which is uniformly closed, which is a ring, and which is closed under inversion (if $f \in \Omega$ and $f > 0$, then $1/f \in \Omega$). In particular, each space $C(X)$ of all continuous functions on a topological space is a complete ordinary function system and if Ω is a linear lattice containing the constant functions, then $B_1(\Omega)$ is a complete ordinary function system (Mauldin, [7, Theorem 8]).

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