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# LOCAL FIXED POINT INDEX THEORY FOR NON SIMPLY CONNECTED MANIFOLDS[1]

BY

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## 1. Introduction

This paper is a sequel to [1]. There we associated to a globally defined map $f: M \to M$ on a compact manifold an obstruction class $o(f) \in H^m(M; \mathscr{B}(f))$, $m = \dim M$, where $\mathscr{B}(f)$ is an appropriate bundle of groups on $M$, with local group isomorphic to $\mathbf{Z}[\pi]$, $\pi = \pi_1(M)$. We also identified $o(f)$ with an element $\mathscr{L}_\pi(f) \in ZR[\pi, \varphi]$, where $R[\pi, \varphi]$ is the set of Reidemeister classes of $\pi$ induced by the homomorphism $\varphi = f_\#: \pi \to \pi$. $\mathscr{L}_\pi(f)$ had the form

$$\mathscr{L}_\pi(f) = \pm \sum_{\rho \in R} I(\rho)\rho$$

where $R = R[\pi, \varphi]$ and $I(\rho)$ is the index of the Nielsen class of $f$ corresponding to $\rho$. This gave us a specific relationship between the obstruction $o(f)$ and the Nielsen number $n(f)$ of $f$, or, more precisely, between $o(f)$ and a *generalized Lefschetz number* $\mathscr{L}_\pi(f)$ which played the role of a global index and which, in turn, was expressible in terms the Nielsen classes of $f$. As a consequence, for example, $\mathscr{L}_\pi(f) = 0$ forces $o(f) = 0$ and one obtains the appropriate converse of the Lefschetz Fixed Point Theorem for non-simply connected manifolds.

Our objective here is to carry out this program locally and thereby give a generalized local index theory.

Section 2 is devoted to the local obstruction index. Starting with a smooth or $PL$ manifold $M$, $\dim M \geq 3$, the inclusion map $M \times M - \Delta \hookrightarrow M \times M$ is replaced by a fiber map $p: E \to M \times M$ and the bundle $\mathscr{B}$ of coefficients is the local system $\pi_{m-1}(F)$ on $M \times M$, where $F$ is the fiber of $p$. The group $\pi_{m-1}(F)$ is identified in [1] as $\mathbf{Z}[\pi]$, where $\pi = \pi_1(M)$ and the action of $\pi \times \pi$ on $\mathbf{Z}[\pi]$ is given by the right action

$$\alpha \circ (\sigma, \tau) = (\operatorname{sgn} \sigma)\sigma^{-1}\alpha\tau.$$

Now, we suppose that we are given a map $f: U \to M$, which is *compactly fixed* on $U$ (i.e. Fix $f$ is compact), $U$ an open set in $M$. Let $\mathscr{B}(f)$ denote the bundle of groups on $U$ induced from $\mathscr{B}$ by $i \times f: U \to M \times M$. The *local obstruction index*

$$o(f) = o(f, U) \in H_c^m(U; \mathscr{B}(f))$$

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