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QUILLEN'S $\mathscr{K}$-THEORY AND ALGEBRAIC CYCLES ON ALMOST NON-SINGULAR VARIETIES

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Introduction

Let X denote an irreducible quasi-projective variety defined over an algebraically closed field, x_0 a distinguished closed point of X. We say that (X, x_0) is almost non-singular if $X - x_0$ is non-singular, and make this assumption in the following discussion.

Let X_i be the set of points (i.e., irreducible cycles) of codimension i in X and let $X_i^* = \{x \in X_i : x_0 \notin \bar{x}\}$. Set

$$C^i = \coprod_{x \in X_i} \mathbf{Z}_x \qquad \text{and} \qquad C^{*i} = \coprod_{x \in X^*_i} \mathbf{Z}_x.$$

Define R^i to be the subgroup of C^i which is generated by the elements of the form (s, f), where s is in X_{i-1}, f is an element of $k(s)^*$, the group of invertible elements in the function field of s, and (s, f) denotes the cycle $((f)_0 - (f)_\infty)$ computed on X. We refer to the elements of R as "relations". The group $C^i/R^i = CH^i(X)$ is the ith graded part of the covariant Chow group (cf. [2]).

Quillen [5] has associated sheaves $\mathscr{K}_{iX}$ with any scheme X, and proved that if X is a non-singular quasi-projective variety then

$$CH^i(X) \simeq H^i(X, \mathscr{K}_{iX}). \tag{0.1}$$

If X is any variety, $H^1(X, \mathscr{K}_{1X})$ still has a geometric interpretation, indeed $\mathscr{K}_{1X} = \mathscr{O}_X^*$; therefore $H^1(X, \mathscr{K}_{1X}) = \operatorname{Pic}(X)$. It is a natural question to inquire about the geometrical meaning of the groups $H^i(X, \mathscr{K}_{iX})$.

Define R^{*i} to be the subgroup of C^{*i} generated by the relations (s, f) with the further requirement $s \in X^*_{i-1}$, i.e., by the relations which avoid the distinguished point. Set $CH^i(X, x_0) = C^{*i}/R^{*i}$. Our interpretation is:

(0.2) THEOREM. *If X is almost non-singular then*

$$CH^i(X, x_0) \simeq H^i(X, \mathscr{K}_{iX}) \quad i > 1.$$

Note that if X is non-singular, (0.1) and (0.2) together provide a highbrow proof that $CH^i(X) \simeq CH^i(X, x_0)$, $i > 1$.

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