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A GHASTLY GENERALIZED n-MANIFOLD[1]

BY

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1. Introduction

A standard source for generalized n-manifolds (finite dimensional ANR's X such that $H_*(X, X - pt; Z) \cong H_*(E^n, E^n - pt; Z)$) is decomposition spaces of cell-like but not necessarily cellular decompositions of n-manifolds. The generalized 3-manifolds that contain no 2-disk constructed by Bing-Borsuk [3] arise as decompositions of S^3 whose nondegenerate elements form a null sequence of noncellular arcs. The improvements produced by S. Singh [14, $n = 3$], and D. Wright [18, $n \geq 4$], in which generalized n-manifolds are constructed containing no proper ANR's of dimension greater than or equal to 2, also result from decompositions of S^n whose nondegenerate elements form a null sequence of noncellular arcs. Cannon-Daverman [5] constructed cell-like totally noncellular decompositions of n-manifolds and used these to build totally wild flows. We add to this list of generalized manifolds having properties not satisfied by honest manifolds and produce a generalized n-manifold X ($n \geq 3$) such that, for each map $F: B^2 \to X$ where $F|S^1$ is an embedding, $F(B^2)$ has nonempty interior in X. An interesting feature of these examples is that, while they arise from totally noncellular decompositions similar to those of [5], they exhibit properties similar to those exhibited in [3], [14], [15] and [18]. Clearly they contain no 2-disks nor ANR's of dimension strictly between 1 and n.

Prior to the description of these examples, this paper lays a broad theoretical groundwork for dealing with defining sequences. It begins by setting forth an axiom base for a general definition of defining sequence for an upper semicontinuous decomposition, more general than the *ad hoc* definition given in [5], adds another axiom for working with cell-like decompositions, introduces the notion of shrinkable defining sequences, and treats the naturality of each. To summarize the remaining contents of the paper, the examples themselves then are constructed in Section 5, their pathology is studied in Section 6, and the result that, with extra care about their construction, their product with E^1 is a manifold is established in Section 7.

Spaces are always assumed to be metrizable and are usually assumed to be locally compact. The general development of defining sequences for decompositions and the specialization to those yielding cell-like decompositions are

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