ILLINOIS JOURNAL OF MATHEMATICS
Volume 26, Number 2, Summer 1982

EQUIVARIANT BUNDLES

BY

R. K. LASHOF

We develop a theory of equivariant bundles, i.e., bundles with a compact Lie group G of automorphisms. Equivariant vector bundles were discussed by Wasserman [9] and Atiyah-Segal [8]; Bierstone [1] considered smooth equivariant bundles; and in [6] we sketched the general theory of equivariant bundles with a finite group G of automorphisms. However, general equivariant bundles are needed in equivariant smoothing theory [5]; and unfortunately, none of the above expositions generalizes without important modifications. As in [6], we generalize the Dold numerable bundle theory [3] to the equivariant case.

1. Numerable G-bundles

Let $p\colon E \to X$ be a locally trivial bundle with fibre F and structure group A. We call p a *G-bundle*, or more precisely a G-A bundle if E and X are G-spaces, p is a G-map, and G acts on E through A-bundle maps. Two G-A bundles over X are called *G-A equivalent* if they are A-equivalent via a G-equivariant map.

Example 1. A G-vector bundle [8] of dimension n is a G-L_n bundle, L_n the group of linear isomorphisms of R^n.

If $p\colon E \to X$ is a G-A bundle, the action of G induces an action of G on the associated principal A-bundle P, again through A-bundle maps. That is, G acts on the left and A acts on the right of P and these actions commute. Conversely, if $p\colon P \to X$ is a principal G-A bundle and A acts on the left of F, then $E = P \times_A F$ is a G-A bundle with fibre F. Two G-A bundles with fibre F are G-A equivalent if and only if their associated principal G-A bundles are G-A equivalent.

In order to prove a covering homotopy property or to produce a classifying space for ordinary bundles, the local triviality condition is essential. Bierstone [1] pointed out that for equivariant bundles one needs a G-local triviality condition for the same purpose. Before defining this condition we recall the local structure of a completely regular G-space X (see [2]): For any $x \in X$ there is a G_x-invariant subspace V_x containing x, called a *slice through* x, such that

$$\mu\colon G \times_{G_x} V_x \to X, \quad \mu[g, v] = gv$$

Received May 22, 1980.

[1] By abuse of notation, we shall write $E_0 = E/X$.