ILLINOIS JOURNAL OF MATHEMATICS
Volume 26, Number 3, Fall 1982

NORMAL N.E.C. SIGNATURES

BY

E. BUJALANCE

1. Introduction

By a non-Euclidean crystallographic (N.E.C.) group, we shall mean a discrete subgroup Γ of isometries of the non-Euclidean plane with compact quotient space, including those reverse orientation, reflections and glide-reflections.

Let G denote the group of isometries of the upper-half plane D and let G^+ denote the subgroup of index 2 in G consisting of the conformal homeomorphisms. If Γ is an N.E.C. group we let $R(\Gamma, G)$ denote the set of isomorphisms $r\colon \Gamma \to G$ with the property that $r(\Gamma)$ is discrete and $D/r(\Gamma)$ is compact. $r_1, r_2 \in R(\Gamma, G)$ are called equivalent if for all $\gamma \in \Gamma$, $r_1(\gamma) = gr_2(\gamma)g^{-1}$ for some $g \in G$. The quotient space is denoted by $T(\Gamma, G)$, the Teichmüller space of Γ. It is homeomorphic to a cell of dimension $d(\Gamma)$. If Γ is a Fuchsian group with signature $(g;\, +;\, [m_1 \cdots m_\iota])$ then $d(\Gamma) = 6g - 6 + 2\iota$. Singerman [3] states that if Γ is a proper N.E.C. group, then $d(\Gamma) = \frac{1}{2}d(\Gamma^+)$.

Macbeath and Singerman [2] have proved that Mod (Γ) fails to be effective in its action on $T(\Gamma, G)$ if and only if there is an N.E.C. group Γ_1 with dim $\Gamma_1 =$ dim Γ and an embedding $\alpha\colon \Gamma \lhd \Gamma_1$.

We shall find here the signatures of all N.E.C. groups Γ such that Mod (Γ) fails to be effective in its action on $T(\Gamma, G)$. For this to be done we shall define in (Section 3) the concept of normal signature. The computation of these normal signatures on N.E.C. groups allows us to solve the problem (Section 4).

The corresponding problem for Fuchsian groups, was essentially solved in the work of Singerman [4].

2. N.E.C. signatures

N.E.C. groups are classified according to their signature. The signature of an N.E.C. group Γ is either of the form

$$(*) \qquad (g;\, +;\, [m_1 \cdots m_\iota];\, \{(n_{11} \cdots n_{1s_1}) \cdots (n_{k1} \cdots n_{ks_k})\})$$

or

$$(**) \qquad (g;\, -;\, [m_1 \cdots m_\iota];\, \{(n_{11} \cdots n_{1s_1}) \cdots (n_{k1} \cdots n_{ks_k})\})$$

The numbers m_i are the periods and the brackets $(n_{i1} \cdots n_{is_i})$ the period cycles.

Received November 14, 1980.