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CERTAIN SECONDARY OPERATIONS THAT DETECT INCOMPRESSIBLE MAPS

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Introduction

Let $f\colon X \to K(Z_2, n)$ be a non-trivial map and let $\operatorname{im} f^*$ be the subset of $H^*(X; Z_2)$ induced by f. If all primary Steenrod operations act trivially on $\operatorname{im} f^*$ then the incompressibility of such maps cannot be detected by such primary operations. In this paper it is shown that if $H^*(X; Z_2)$ satisfies certain conditions, there is a sequence of secondary operations that can be used to detect the incompressibility of f. These secondary operations are of the form $Sq^{Q_k}\Phi$ where Φ is a secondary operation and $Q_k = (q_k, \ldots, q_1)$ is an admissible sequence.

Let Φ be a secondary operation associated to the relation $\alpha \circ \beta = 0$. If $x = f^*(\iota_n)$ and $\Phi(x)$ is defined, there is a fibration

$$\Omega K_1 \xrightarrow{i} E \xrightarrow{\pi} K(Z_2, n),$$

a map $\tilde{f}\colon X \to E$ and an element $w \in H^*(E; Z_2)$ such that $\Phi(x) = \tilde{f}^*(w)$ mod indeterminacy. The following are proved:

THEOREM. *Let $f\colon X \to K(Z_2, n)$ classify $x \in H^n(X; Z_2)$, $Q_k = (q_k, \ldots, q_1)$ be an admissible sequence and Φ be a secondary operation associated with $\alpha \circ \beta = 0$ and defined on x. If for all positive integers k,*

(a) $Sq^{Q_k}(i^*(w)) = \sum_j Sq^{p_{j,k}} v_{j,k}$ *where* $0 < p_{j,k} \le q_k$ *for all* j,
(b) $v_{j,k}$ *transgresses to a non-zero element for all* j,
(c) $M_k - n < q_k \le M_k$ *where* $M_k = \deg w + \sum_{i=1}^{k-1} q_i$,
(d) $Sq^{Q_k}\Phi(x) \ne 0 \bmod (\operatorname{im} Sq^{Q_k}\alpha + \sum_{j,k} \operatorname{im} Sq^{p_{j,k}})$,

then f is incompressible.

COROLLARY. *Let $\pi_n\colon E_n \to K(Z_2, n)$ be the universal fibration classifying $x \in H^n(X; Z_2)$ for which x is annihilated by A_2, the* mod 2 *Steenrod algebra. π_n is incompressible.*

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