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THE AUTOMORPHISMS OF $PU_4^+(K, f)$

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The automorphisms of the classical groups have been discussed in many places [1], [2], [3], [6], [7], [9], [10]. The paper presents a solution to the problem in a minimal case heretofore not discussed. Namely, let E be a vector space over a field K admitting an automorphism J such that $J^2 = 1$. Let there be a hermitian sesquilinear form $f\colon E \times E \to K$ defined relative to J. Then, designate by $U(E, f)$ the group of linear transformations leaving invariant f. When $\dim_K E = n$, we also use the notation $U_n(K, f)$. Let $Z(U_n(K, f))$ designate the center of $U_n(K, f)$. Then $U_n(K, f)/Z(U_n(K, f))$ acts on the projective geometry $P(E)$ obtained from E. Let $U_n^+(K, f)$ be the subgroup of $U_n(K, f)$ consisting of transformations of determinant 1. Let $PU_n^+(K, f)$ denote the image of $U_n^+(K, f)$ in $PU_n(K, f)$.

A sesquilinear form f is said to be *anisotropic* if its Witt index is zero. Then $f(x, x) = 0$ only if $x = 0$. It is known that f is never anisotropic if K is finite. Also, the group $U_n(K, f)$ contains no unipotent transformations when f is anisotropic. This means that the action of every element of $U_n(K, f)$ on E is completely reducible.

The group $U_n(K, f)$ acting on E is the group of semilinear transformations u acting on E relative to an automorphism $\sigma = \sigma(u)$ of K such that for all x, $y \in E$, $f(ux, uy) = ef(x, y)$ where e is an element of K such that $e^J = e$. It is known that $\Gamma U_n(K, f)$ is the normalizer of $U_n^+(K, f)$ in the group $\Gamma L_n(K)$ of semilinear transformations. Let $P\Gamma U_n(K, f)$ denote its image in the group $P\Gamma L_n(K)$ of collineations of $P(E)$. Then $P\Gamma U_n(K, f) \subseteq \text{Aut } PU_n^+(K, f)$. When $n \geq 3$, it is known that $P\Gamma U_n(K, f) = \text{Aut } PU_n(K, f)$ except when $n = 4$ and f is anisotropic, the case we treat in this paper.

Indeed, the most conclusive results in this direction are due to Wonenberger [10] who covered the cases when $n \neq 4$ and K has characteristic not 2, and Borel and Tits [1] who in a very general argument worked out the automorphisms of almost simple algebraic groups defined over K when the groups contain unipotent elements. This covers the case $PU_4^+(K, f)$ except when f is anisotropic. The result of this paper is the following.

THEOREM. *Let f be an anisotropic hermitian sesquilinear form defined over an infinite field of characteristic not* 2, *relative to an automorphism J of K of order* 2. *Then* Aut $PU_4^+(K, f) = P\Gamma U_4(K, f)$.

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