ILLINOIS JOURNAL OF MATHEMATICS
Volume 27, Number 2, Summer 1983

# THE HOMOLOGY OF THE JAMES-HOPF MAPS

BY

NICHOLAS J. KUHN

If $X$ is a path connected space, there are filtered spaces $C_nX$ and $CX$ which approximate $\Omega^n\Sigma^nX$ and $QX = \lim_{\to} \Omega^n\Sigma^nX$ respectively [11]. Quotients of successive filtrations are the extended power spaces denoted by $D_{n,q}X$ and $D_qX$. Snaith [13], generalizing a result of Kahn [6], showed that

$$\Sigma^\infty\Omega^n\Sigma^nX \simeq \bigvee_{q\geq 1} \Sigma^\infty D_{n,q}X$$

where $\Sigma^\infty Y$ denotes the suspension spectrum of a space $Y$.

Projection onto the $q$-th wedge summand and adjunction yield the James-Hopf maps

$$j_q : QX \to QD_qX \quad \text{and} \quad j_q : \Omega^n\Sigma^nX \to QD_{n,q}X.$$

It is the purpose of this paper to study the induced maps $j_{q*}$ on homology and from this deduce geometric results. All homology will be with $Z_2$ coefficients.

Our geometric input is the following. Let $D_0X = S^0$. In [2] it is shown that, via passage to quotients of filtrations, the additive $E_\infty$-structure on $CX$ induces an $E_\infty$-ring structure on $\Pi_{q\geq 0} QD_qX$. Let $j_0 : QX \to QD_0X$ send everything to 1, the identity map in $QS^0$. The maps $j_q$ piece together to give a map

$$j : QX \to \prod_{q\geq 0} QD_qX.$$

F. Cohen, R. Cohen, May, and Taylor [2] show that the map $j$ is exponential in the sense that it takes the additive $E_\infty$ structure on $QX$ to the multiplicative one on $\Pi_{q\geq 0} QD_qX$. Also $j_1 : QX \to QX$ is homotopic to the identity and the composite

$$X \xrightarrow{\eta} QX \xrightarrow{j_q} QD_qX$$

is nullhomotopic for $q > 1$.

Recall that the rich structure of iterated loop spaces allows one to define operations on their homology. Under these operations, $H_*(QX)$ is generated by $H_*(X)$.

Received May 19, 1981.