ILLINOIS JOURNAL OF MATHEMATICS
Volume 27, Number 3, Fall 1983

THE PRODUCT OF TWO OR MORE NEIGHBORING INTEGERS IS NEVER A POWER

BY

JAN TURK

1. Introduction

In 1975 Erdös and Selfridge [1] proved the elegant result: *the product of two or more consecutive positive integers is never a power*. Earlier, Rigge [6] and, independently, Erdös [7] had shown that such a product is never a square.

In this paper we prove a generalisation, namely: *the product of two or more neighboring integers is never a power.*

Here we say that distinct integers are *neighboring* if they belong to a short interval $(N, N + c\log\log\log N)$ for some absolute constant c (and, of course, if $N \leqslant 16$ we interpret $\log\log\log N$ as 1). Our principal result is false for infinitely many N if the interval is lengthened to

$$\exp(12(\log N \log\log N)^{1/2}),$$

and is false for all N if the interval is as long as $cN^{1/2-\varepsilon}$ for certain positive constants c, ε.

Actually, we consider a more general situation. Our products of neighboring integers allow for repetition, so our statement becomes that *the product of two or more neighboring integers, allowing repetition, is never, other than trivially, a power*. Moreover, we deal with "almost powers" rather than "perfect powers." This is to say: we consider quantities of the shape ab^m with the integer a "small" relative to b^m and we actually show that the quantities we consider are never (other than trivially) "almost powers."

Finally we remark on what constitutes "triviality." We consider finite products

$$\Pi n_i^{m_i},$$

with the n_i in the given "short interval," and consider such products "not trivially a possible almost *m-th* power" if $\gcd(m,m_i) = 1$ for each i. It would seem more natural to ask of the m_i that not all the m_i be multiples of m. But this raises considerable difficulties: though Tijdeman [8] has shown that two sufficiently large *consecutive* integers cannot both be powers,

Received June 1, 1981.