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ON THE HOMOTOPY TYPE OF DIFFEOMORPHISM GROUPS[1]

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Introduction

Let M be a closed smooth manifold and $Diff_0(M)$ the identity component of the group of C^∞ diffeomorphisms of M. We are concerned here with the way in which the homotopy type of $Diff_0(M)$ depends on the smooth structure of M. Our principal result along these lines states that, if M_1 and M_2 are homeomorphic smooth manifolds, then, for suitable subrings Λ of the rationals Q (obtained from the integers Z by inverting a finite set of primes), $Diff_0(M_1)$ and $Diff_0(M_2)$ have the same Λ-homotopy type. (Recall that two nilpotent spaces X and Y are said to have the same Λ-homotopy type if there is a space W and mappings $X \to W$ and $Y \to W$ inducing isomorphisms

$$\pi_q(X) \otimes \Lambda \simeq \pi_q(W) \otimes \Lambda, \qquad \pi_q(Y) \otimes \Lambda \simeq \pi_q(W) \otimes \Lambda$$

for all $q \geq 0$. See [1].) In particular, we define an integer $\nu = \nu(M_1, M_2)$ in Section 1 depending only on bundle data associated to M_1 and M_2 such that the following holds:

THEOREM. *Let M_1 and M_2 be homeomorphic smooth n-manifolds, $n \neq 4$, and let Λ be the subring of Q obtained from Z by inverting $\nu(M_1, M_2)$. Then $Diff_0(M_1)$ and $Diff_0(M_2)$ have the same Λ-homotopy type.*

We prove an analogous result for the (simplicial) group **PL**(M) of PL-homeomorphisms of a PL-manifold (Theorem 1.3). We also prove a similar result regarding the discrete group homology (with coefficients in Λ) of $Diff_0(M_1)$ and $Diff_0(M_2)$ (Theorem 1.2).

Another type of result that we investigate involves the mapping of the diffeomorphism group of a smooth manifold onto its frame bundle. Let M be a smooth closed n-manifold and let $P(M)$ be the frame bundle of M; that is, the principal $GL(n, R)$ bundle associated with the tangent bundle of M. Then $Diff_0(M)$ acts on $P(M)$ and we can define a mapping $\sigma: Diff_0(M)$

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