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# PREDICTION FROM PART OF THE PAST OF A STATIONARY PROCESS

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## 1. Introduction

Let $w$ be the spectral density of a stationary process $X(t)$ $(-\infty < t < \infty)$. It will be assumed that $(\log w)/(1 + x^2)$ is integrable on $R$ with respect to Lebesgue measure. Thus $w(x) = |h(x)|^2$ where $h$ is an outer function in $H^2$, the Hardy space for the upper half-plane. Let $Z$ denote the space of measurable functions which are square-integrable against the measure $w(x)dx$, and let $Z(a, b)$ denote the closed subspace of $Z$ generated by $\{e_t : a \leq t \leq b\}$ $(e_t(x) \equiv e^{itx})$. The obvious meanings will be ascribed if $a$ or $b$ is $\pm\infty$. The problem studied here is that of approximating orthogonal projection on $Z(-a, a)$. In [5], Dym and McKean worked out a recipe for projection on $Z(-a, a)$, but their solution is difficult to apply. A less general approach was adopted by Segier [10] to work out a projection formula in the case $w = |P|^2/|B|^2$ where $P$ is a polynomial and $B$ is an entire function of finite exponential type. The latter approach will be followed here: under mild assumptions, $Z(-a, a) = Z(-a, \infty) \cap Z(-\infty, a)$, so the desired projection may be approximated by "projecting back and forth" on $Z(-a, \infty)$ and $Z(-\infty, a)$; projection onto these last subspaces is straightforward. How good this scheme is depends upon structural properties of the weight $w$. This is discussed in Sections 3 and 4; a connection between these approximations and strong mixing is given in Section 5.

## 2. Preliminaries

Let $L^2$ denote the Hilbert space of functions on $R$ which are square-summable with respect to Lebesgue measure. Then the map $S: f \to hf$ is an isometry of $Z$ into $L^2$. Moreover, since $h$ is outer, $S$ is surjective (See [5], p. 97) and maps $Z(-\infty, a)$ and $Z(a, \infty)$ respectively onto $(e_a h/\bar{h})\overline{H^2}$ and $e_a H^2$ (where the bar denotes complex conjugation). The following notation will be used:

(1) $P_a$ is projection onto $e_a H^2$ in $L^2$;
(2) $Q_a$ is projection onto $(e_a h/\bar{h})\overline{H^2}$;

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