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SPHERES AND CYLINDERS: A LOCAL GEOMETRIC CHARACTERIZATION

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This note characterizes the smooth hypersurfaces in Euclidean space which satisfy locally the following geometric condition.

(*) For each two points of the surface, the chord joining them meets the normal to the surface in equal angles at the two points.

This condition arose in the study of the Bochner-Martinelli integral formula, which is a higher-dimensional analogue of Cauchy's integral formula in the complex plane. In [1] the author proved that the Bochner-Martinelli operator, viewed as a bounded singular integral operator acting on the Hilbert space of square-integrable functions on the boundary of a smooth bounded domain, is self-adjoint if and only if the domain is a ball. The proof hinged on the following geometrical result, which has nothing to do with complex analysis.

GLOBAL CHARACTERIZATION THEOREM. *Let G be a bounded C^1 smooth domain in $R^k, k \geq 2$. Then the boundary of G satisfies (*) if and only if G is a ball.*

The proof of the Global Characterization Theorem in [1] uses a compactness argument, and therefore does not address the question, asked by N. Kerzman, of which surfaces satisfy (*) locally. It is easy to see that, in addition to subsets of spheres S^n, subsets of planes R^n and subsets of spherical cylinders $S^n \times R^m$ do satisfy (*). Our result in this article is that there are no other possibilities.

LOCAL CHARACTERIZATION THEOREM. *Let M be a connected smooth C^1 local hypersurface in R^k, $k \geq 2$. Then M satisfies (*) if and only if M lies in a surface of the form*

$$S^{k-1-j} \times R^j, \quad 0 \leq j \leq k-1.$$

Our hypotheses are, more explicitly, that there is an open set $U \subset R^k$ and a continuously differentiable normalized defining function

$$r: U \to R$$

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