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POTENTIAL THEORY ON COMPLEX PROJECTIVE SPACE: APPLICATION TO CHARACTERIZATION OF PLURIPOLAR SETS AND GROWTH OF ANALYTIC VARIETIES

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0. Introduction

A set $E \subset \mathbf{P}^n\mathbf{C}$ is said to be locally pluripolar if for each point $p \in E$ there exists a neighborhood U of p and a plurisubharmonic function ψ defined on U such that $E \subset U \quad \{x : \psi(x) = -\infty\}$ and ψ is not identically $-\infty$ on each component of U. A basic problem in function theory of several complex variables is to characterize those sets which are pluripolar. In his paper on projective capacity [1], Alexander gives a characterization of pluripolar sets in $\mathbf{P}^n\mathbf{C}$ in terms of a Tchebycheff constant $\tau(E)$. His theorem says that E is locally pluripolar if and only if $\tau(E) = 0$. The constant $\tau(E)$ is defined in terms of normalized homogeneous polynomials on $\mathbf{P}^n\mathbf{C}$. Another characterization of pluripolar sets was recently given by Bedford and Taylor [3]. Their characterization involves the Monge-Ampere equation and a "balayage" for a set $E \subset \mathbf{C}^n$.

In this paper I give a characterization of locally pluripolar sets in $\mathbf{P}^n\mathbf{C}$ in terms of a singular integral with respect to a probability measure, supported on E; the set in question. The kernel of this singular integral is defined on

$$\mathbf{P}^n\mathbf{C} \times \mathbf{P}(S_{n+1,d})$$

where $S_{n+1,d}$ is the d-fold symmetric tensor product of $\mathbf{C}^{n+1}$; hence the kernel is not symmetric. Explicitly the kernel is given by

$$K_d(Z, a) = \log \frac{|Z|^d}{|a^*(Z)|}$$

where a^* denotes the homogeneous polynomial of degree d dual to a.

The kernel $K_d(Z, a)$ also turns out to play an important role in value distribution theory. If X is an analytic subvariety of $\mathbf{C}^n$ then a basic problem is to relate the growth of X to the growth of intersections of X with algebraic subvarieties of $\mathbf{C}^n$. This was done in [9] in the case where the algebraic subvarieties where hyperplanes. We also remarked in [9] that the growth of X could be related to the growth of $X \cap V^\lambda$ where $\{V^\lambda\}$ was a sufficiently large family of

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