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TEMPERED, INVARIANT, POSITIVE-DEFINITE DISTRIBUTIONS ON $SU(1,1)/\{\pm 1\}$

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1. Introduction

Let G denote the group of conformal mappings of the interior of the unit circle, a Lie group which is naturally isomorphic to both $SU(1, 1)/\{\pm 1\}$ and $SL(2,\mathbf{R})/\{\pm 1\}$. In this paper we establish, via the Fourier transform, a bijective correspondence between the collection of tempered, invariant, positive-definite distributions on G and the easily defined class of tempered Bochner measure pairs. Viewed in another way, the result shows that tempered, invariant, positive-definite distributions are merely integrals, in the distributional sense, of characters of the principal and discrete series representations of G.

The major tools used in this work are the various isomorphisms which are obtained via the operator Fourier transform on G. For each $1 \leq p \leq 2$ let $\mathscr{C}^p(G)$ be Harish-Cchandra's L^p-Schwartz space, with $\mathscr{C}(G) = \mathscr{C}^2(G)$. In his Ph.D. dissertation [1] Arthur characterized the image of $\mathscr{C}(G)$ under the Fourier transform for G any semi-simple Lie group of real rank one. However, an invariant, positive-definite distribution is not, in general, tempered; i.e., it does not extend to a continuous linear functional on $\mathscr{C}(G)$. Such distributions extend, instead, onto $\mathscr{C}^1(G)$ [4, §4]. Unfortunately, for $1 \leq p < 2$, the Fourier transform image of $\mathscr{C}^p(G)$ has yet to be determined, even for $SU(1,1)/\{\pm 1\}$. Given the importance of such results for our work, in this paper we will confine ourselves to the tempered distributional case.

In §§4-6 of this paper we state Arthur's Theorem for $SU(1, 1)/\{\pm 1\}$, and develop certain important results concerning spherical function spaces and their images under the Fourier transform. In §7 tempered invariant distributions are examined. It is shown that such a distribution T is determined, via the spherical decomposition of the Fourier transform, by the zonal spherical transform $\hat{T}$ and a unique complex counting measure μ_d (Theorem 7.4). In §8 it is shown that if T is also positive-definite, then $\hat{T}$ is given by a measure μ_c on $\mathbf{R}$, and both μ_c and μ_d are non-negative and of polynomial growth. In fact, there is a bijection between the collection of tempered, invariant, positive-definite distributions and the collection of pairs (μ_c, μ_d) (Theorem 8.2). In §9 this last result is reformulated to show that a tempered, invariant, positive-definite distri-

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