1. Let $K$ be a field with a non-trivial non-Archimedean valuation $|\cdot|$. Let $E$ be a Banach space over $K$ with norm $\|\cdot\|$. The unit ball $V = \{\lambda \in K : |\lambda| \leq 1\}$ is the valuation ring of $K$. Let $E$ be a module over this ring. A nonempty subset $A$ of $E$ is called absolutely convex if it is a $V$-module of $E$; that is, if $a, b \in A$ and $\lambda, \mu \in V$, then $\lambda a + \mu b \in A$. A coset of an absolutely convex subset is said to be convex. A subset $A$ of $E$ is said to be compactoid if for every $\varepsilon > 0$ there exists a finite set $X \subseteq E$ such that $A \subseteq \{x \in E : \|x\| \leq \varepsilon\} + \overline{C}_oX$, where $\overline{C}_oX$ denotes the closed convex hull of $X$ (A. van Rooij [5], p. 134). The problem which we consider in this section is the following.

Let $A$ and $B$ be closed convex subsets of $E$. Under what circumstances is the subset $A + B$ closed? It is well known that if $A$ is compact, then $A + B$ is closed. Further, A. van Rooij [5] has shown that if $K$ is spherically complete and $A$ is compactoid, $A + B$ is closed. By applying the results in this section to continuous linear operators, we can obtain Banach’s closed range theorem and the Fredholm alternative theorem in non-Archimedean Banach space. In L. Narici, E. Beckenstein and G. Bachman [3, p. 91], the Fredholm alternative theorem is mentioned for the completely continuous operator. In Section 3, we shall extend it to compact operators as defined by A. van Rooij [6, p. 142]. The existence of the nonzero completely continuous linear operator implies that $K$ is locally compact. However, even if $K$ is not locally compact, there exists a nonzero compact linear operator of $E$ to $F$, when $E$ and $F$ are Banach spaces [6, p. 182].

First we show the following result.

**Lemma 1.** Let $A$ and $B$ be subsets of $E$. If $A$ is open and convex, then $A + B$ is closed. In particular, every open convex subset of $E$ is closed.