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M-IDEALS OF L^∞/H^∞ AND SUPPORT SETS

BY

RAHMAN M. YOUNIS[1]

1. Introduction

Let L^∞ be the usual space of bounded measurable functions on the unit circle T. Let H^∞ denote the subalgebra of L^∞ consisting of functions on T that are radial limits of bounded analytic functions of the open unit disk, and $H^\infty + C$ denotes the closed linear span of H^∞ and C, where C is the space of continuous functions on T. The norm of an L^∞-function f is denoted by $\|f\|$. If $H^\infty \subseteq A \subseteq L^\infty$, we let $M(A)$ denote the maximal ideal space of A. Elements of A may be identified with functions on $M(A)$. Such an algebra is commonly called a Douglas algebra.

If E is a generalized peak set for H^∞, we define

$$H_E^\infty = \{ f \in L^\infty \colon f_{|E} \in H_{|E}^\infty \}.$$

The algebra $(H^\infty + C)_E$ is defined analogously. If E is a generalized peak set for $H^\infty + C$, then $(H^\infty + C)_E$ is closed. These algebras appeared in [16] and [11]. The reader is referred to [5], [3] and [9] for the theory of uniform algebras and to [6] and [13] for the general basic facts about H^∞.

If A is a closed subalgebra of $C(X)$, X is a compact space, then the essential set of A is the zero set of the largest closed ideal of $C(X)$ which lies in A. Equivalently, it is equal to $\overline{\bigcup \operatorname{supp} \mu}$, where $\mu \in A^\perp$.

The concept of M-ideals has been used by the authors of [10], [11], [16] and [17] in order to prove that L^∞/A is an M-ideal in L^∞/H^∞ for a certain Douglas algebra A. A subspace K of a Banach space Y is called an M-ideal of there exists an L-projection P from Y^* onto $K^\perp$, that is, P is a projection such that $\|y\| = \|Py\| + \|y - Py\|$ for all $y \in Y^*$. If K is an M-ideal of Y and if $x \in Y$ then there exists $m \in K$ such that $\operatorname{dist}(x, K) = \|x - m\|$ [1]. If $x \in Y \setminus K$ then

$$\operatorname{span}\{ m \colon m \in K, \operatorname{dist}(x, K) = \|x - m\| \} = K \quad [7].$$

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